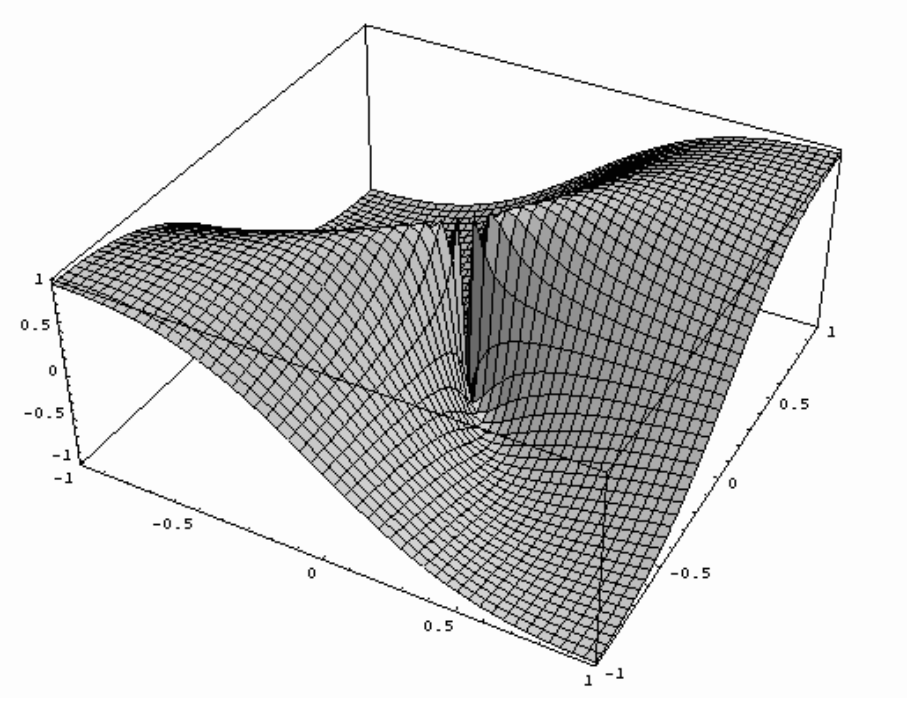
TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – II Real Analysis



COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

TRB – PG MATHS
REAL ANALYSIS

Defn.: Cardinal Number

The number of elements in the set is called cardinal number. It is denoted by $n(A)$.

E.g: If $A=\{a, e, i, o, u\}$ then $n(A) = 5$.

Results:

① In any non-empty set the cardinal number is an +ve integer.

② The cardinal number of finite set is finite.

③ The cardinal number of empty set is zero. (ie) $n(\phi) = 0$.

Countable set and Uncountable set

Defi.: Equivalent Set

Two sets A and B are said to be equivalent if there exists a bjection f from A to B. It is denoted by $A \sim B$.

Defi.: Countable (or) Denumerable

A set A is said to be countable (or) Denumerable if 1-1 and onto relation between A and the set of natural numbers N.

Defi.: Uncountable (or) Non-denumerable.

A set which is not countable is called uncountable.

E.g: ① The set of all integers is countable.

② The set $\{1, 1/2, 2, 1/3, 3, \ldots\}$ is countable.

Scanned by CamScanner

Theorems

1. $\mathbb{Z}$ is countable
2. $\mathbb{Q}$ is countable.
3. Every subset of countable set is countable
4. $\mathbb{Z}^+$ and $\mathbb{Q}^+$ is countable.
5. If A and B are countable sets then $A \times B$ is countable.
6. $\mathbb{N} \times \mathbb{N}$ is countable. Also $\mathbb{Z}^+ \times \mathbb{Z}^+$ and $\mathbb{Q}^+ \times$
7. Countable union of countable set is coun
8. The set of all polynomials $a_0 + a_1 x + a_2 x^2 + \ldots$ where each a_i is a rational no. is countable
9. The set of all rational no. in the inter $[0, 1]$ is countable.

Theorems

1. The set of irrational nos. $\mathbb{Q}'$ is uncountable
2. $\mathbb{R}$ & $\mathbb{C}$ are uncountable.
3. Every interval is uncountable. i.e. $[0, 1]$ & $(0, 1]$ is uncountable.
4. Any interval in $\mathbb{R}$ which contains more than one point is uncountable.

Scanned with CamScanner

TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – V

Operations Research

COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

PG-TRB

PG - MATHEMATICS

OPERATIONS RESEARCH

UNIT - V

Introduction:

The term Operations Research was first Coined by Mcclosky and Trefthen in 1940.

The decision making technique was developed by George B. Dantzig and he formulated the general linear programming problem and developed the Simplex method in 1947.

O.R was first recognized as a subject in universities in 1950.

The main theme of O.R is "winning the war without actually fighting it."

Linear programming:

The word linear indicates that all relationships involved in a particular problem are linear.

General Linear programming problem: (LPP)

Let $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \rightarrow$ (I) (Objective function)

where c_i's are real constants.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq \text{or} \geq \text{or} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq \text{or} \geq \text{or} = b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq \text{or} \geq \text{or} = b_m \end{aligned}\right\} \rightarrow \text{(II)}$$ Constraints

where a_{ij}'s b_j's are real constants

$x_j \geq 0, \quad j = 1, 2, \ldots n \rightarrow$ (III) (non-negative restrictions)

Scanned with CamScanner

The problem of determining an n-tuple $(x_1, x_2, \ldots$ which makes Z a maximum (or minimum) & which satisfies II & III is called the general linear programming problem.

Here, (I) is called as the Objective function.

(II) are called the constraints

(III) are called the non-negative restrictions

Solution: An n-tuple $(x_1, x_2, \ldots x_n)$ of real numbers which satisfies the constraints of a General LPP is called a solution to the general LPP.

Feasible Solution:

Any solution to a General LPP which also satisfies the non-negative restrictions of the problem is called a feasible solution of the general LPP.

(X)

Optimum Solution: (Optimal Solution)

Any feasible solution which optimises (maximizes or minimizes) the objective function of a general LPP is called an optimal solution.

Remark:

The variables $x_1, x_2, \ldots x_n$ are called decision variables.

Standard form of LPP.

Max (or) Min $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$.

Subject to $a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n = b_i$.

$i = 1, 2, \ldots m$, $j = 1, 2, \ldots n$, $x_j \geq 0$.

Here the RHS of each constraints is non-negative [$b_i \geq 0, \forall i$]

Scanned with CamScanner

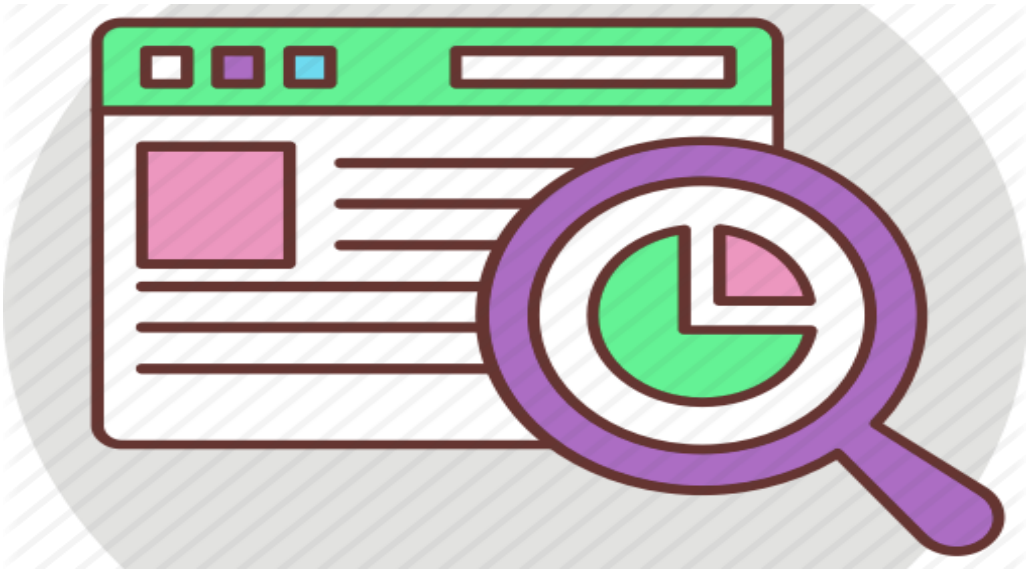
TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – VI

Functional Analysis



COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

PG-TRB_ MATHS

UNIT – VI

FUNCTIONAL ANALYSIS

Normed Linear Space

Defi.: Let N be a complex (or real) linear space. A norm on N is a function $\|\|: N \to R$ such that for all $x, y \in N$ and $\alpha \in C$ (or R).

N_1 : $\|x\| \geq 0$ and $\|x\| = 0$ iff $x = 0$.

N_2 : $\|x+y\| \leq \|x\| + \|y\|$

N_3 : $\|\alpha x\| = |\alpha| \, \|x\|$.

A linear space with a norm $\|\ \|$ defined on it as above is called a normed linear space.

Theorem ① A normed linear space N is a metric space w.r.t. the metric defined by $d(x,y) = \|x-y\|$ $\forall x, y \in N$.

② If N is a normed linear space then

$|\|x\| - \|y\|| \leq \|x-y\|$ for all $x, y \in N$

③ If N is N.L.S., then the norm $\|\|: N \to R$ is a continuous fun. on N.

④ The operation of addition and scalar multi. in N are jointly continuous.

Defi: Let $(N, \|\ \|)$ be a N.L.S. A seq. $\{x_n\}$ in N is said to converge to an element x in N if given $\epsilon > 0$ $\exists$ a +ve integer n_0 s.t. $\|x_n - x\| < \epsilon$ $\forall n \geq n_0$

It follows that $x_n \to x$ iff $\|x_n - x\| \to 0$ as $n \to \infty$.

Scanned with CamScanner

Cauchy Sequence

Defi: A seq. $\{x_n\}$ in N is called a Cauchy seq. in N, if given $\epsilon > 0$ $\exists$ a +ve integer n_0 s.t. $\|x_m - x_n\| < \epsilon$ $\forall$ $m, n \geq n_0$. This means that if $\{x_n\}$ is a Cauchy seq. in N, then $\|x_m - x_n\| \to 0$ as $m, n \to \infty$.

Properties

(1) If N is a N.L.S. every cgt. seq. is a Cauchy seq. But the converse is not true.

(2) If a Cauchy seq. has a cgt. subseq. then the whole seq. is cgt.

(3) Every Cauchy seq. in a N.L.S. is bounded.

Defi: A N.L.S. N is said to be complete if every Cauchy seq. in N cges. to an elt. of N. This means that if $\|x_m - x_n\| \to 0$ as $m, n \to \infty$, then $\exists$ $x \in N$ s.t. $\|x_n - x\| \to 0$ as $n \to \infty$.

Banach Space

Defi: A Complete N.L.S. is called a Banach Space.

In the defi. of a B.S. completeness means that if $\|x_m - x_n\| \to 0$ as $m, n \to \infty$ where $\{x_n\} \subset N$, then $\exists$ a $x \in N$ s.t. $\|x_n - x\| \to 0$ as $n \to \infty$.

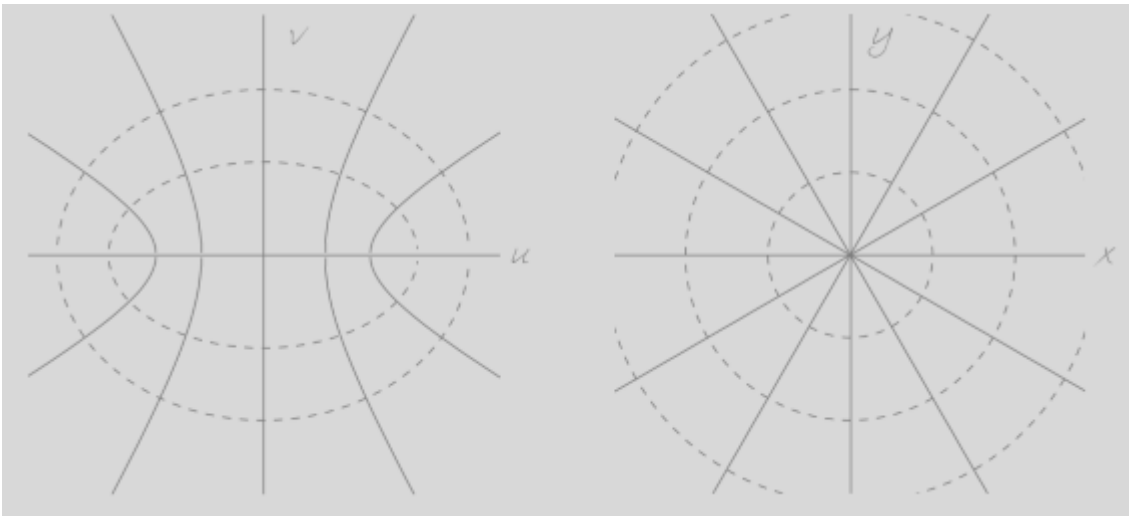
Scanned with CamScanner

TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – VII-Complex Analysis



COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

TRB PG – MATHEMATICS

UNIT-VII – COMPLEX ANALYSIS

Complex Number System

The square of real number is never negative. As such, no real number satisfies the equation $x^2 = -1$. Equations of this type caused the introduction of a new type of numbers called complex numbers.

complex numbers

All ordered pairs (α, β) of all real numbers α & β with equality among them being defined by that

(i) $(\alpha_1, \beta_1) = (\alpha_2, \beta_2) \Rightarrow$ iff $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$ & with two binary operations $+$ and $\cdot$ defined by

(ii) $(\alpha_1, \beta_1) + (\alpha_2, \beta_2) = (\alpha_1 + \alpha_2, \beta_1 + \beta_2)$

(iii) $(\alpha_1, \beta_1) \cdot (\alpha_2, \beta_2) = (\alpha_1 \alpha_2 - \beta_1 \beta_2, \alpha_1 \beta_2 + \alpha_2 \beta_1)$,

are called complex numbers.

Field of complex numbers

↳ That set C of complex numbers is a field.

↳ The element $(0,0)$ is the additive identity and

↳ the element $(1,0)$ is multiplicative identity.

↳ The inverse of (α, β) under addition is $(-\alpha, -\beta)$.

↳ If (x, y) is the multiplicative inverse of (α, β), then

$$(\alpha, \beta) \cdot (x, y) = (1, 0)$$

or $(\alpha x - \beta y, \alpha y + \beta x) = (1, 0)$

This leads to solving the equations $\alpha x - \beta y = 1$ and $\alpha y + \beta x = 0$ for x and y whose values are

Scanned with CamScanner

$$x = \frac{\alpha}{\alpha^2+\beta^2}, \quad y = \frac{-\beta}{\alpha^2+\beta^2}$$

if $\alpha \neq 0$ and $\beta \neq 0$. So the multiplicative inverse of (α, β), when $(\alpha, \beta) \neq (0,0)$ is

$$\left(\frac{\alpha}{\alpha^2+\beta^2}, \frac{-\beta}{\alpha^2+\beta^2} \right)$$

Scalar multiplication of a complex number

The scalar multiplication of a complex number (α, β) by a real number λ, is defined by

$$\lambda(\alpha, \beta) = (\lambda\alpha, \lambda\beta).$$

Note

(1) If z, z_1, z_2 are any three complex numbers, then

a) $|z| = |\bar{z}|$ b) $z\bar{z} = |z|^2$ c) $|z_1 z_2| = |z_1|\,|z_2|$

d) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ e) $|z^n| = |z|^n$.

Inequalities in terms of Moduli (order relations)

There is no order relation in the complex number system

(ie) there is no meaning in saying that $z_1 > z_2$ unless z_1 and z_2 are real.

Thm

If z, z_1, z_2 are any three complex numbers, then

(i) $-|z| \leq Re\, z \leq |z|$, $-|z| \leq Im\, z \leq |z|$.

(ii) $|z_1 + z_2| \leq |z_1| + |z_2|$.

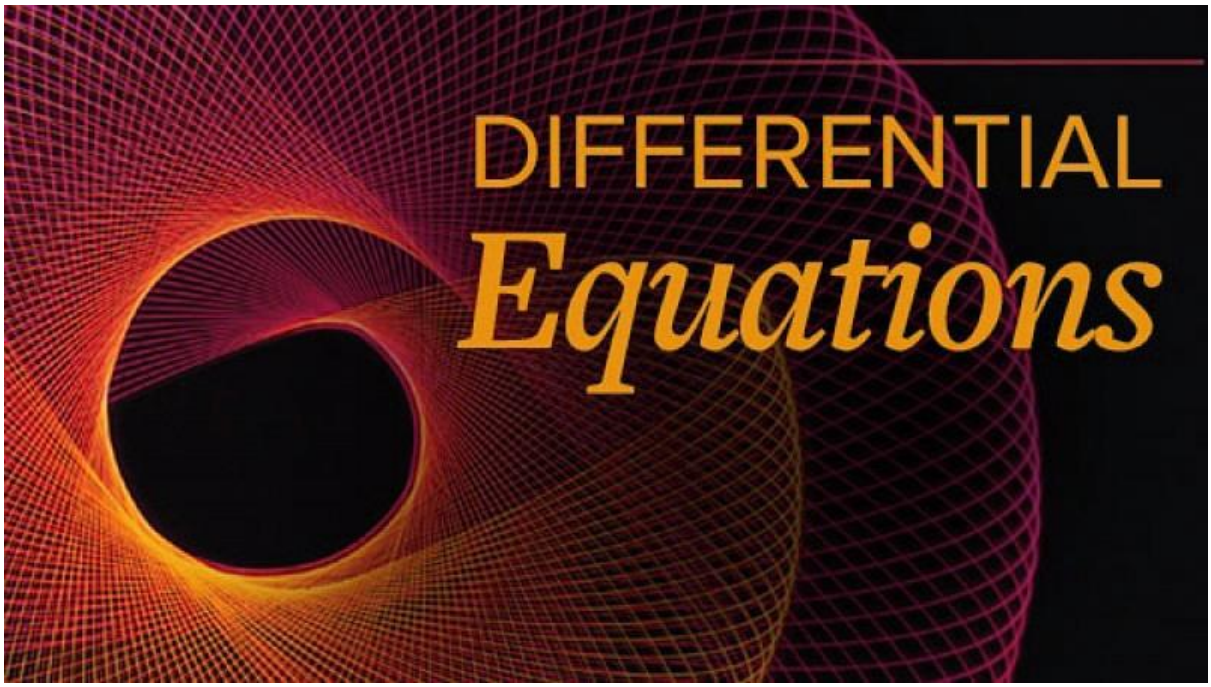
(iii) $|z_1 - z_2| \geq |\,|z_1| - |z_2|\,|$ [the (ii) & (iii) are called triangle

TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – VIII



PG-TRB 2019 – 20

UNIT - VIII

Differential Equations

Linear differential equation – Constant Co-efficients – Existence of Solutions – Wrongskian – Independence of Solutions – Intial value Problems for second order equation – Integration in Series – Bessel's equation – Legendre and Hermite polynomials – elementary Properties – Total differential equations – first order partial differential equation – charpits method.

Basic Concepts Differential formulae:

1) $\frac{d}{dx}(x^n) = n x^{n-1}$
2) $\frac{d}{dx}(e^x) = e^x$
3) $\frac{d}{dx}(\text{Constant}) = 0$
4) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
5) $\frac{d}{dx}(1/x) = -1/x^2$
6) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
7) $\frac{d}{dx}(\sin x) = \cos x$
8) $\frac{d}{dx}(\cos x) = -\sin x$
9) $\frac{d}{dx}(\tan x) = \sec^2 x$
10) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
11) $\frac{d}{dx}(\sec x) = \sec x \tan x$
12) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cos x$
13) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -n x^{-n-1}$
14) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
15) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
16) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
17) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
18) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$
19) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
20) $\frac{d}{dx}(a^x) = a^x \log a$.

Rules:

1) $\frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x))$
2) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
3) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
4) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
5) If $y = y(t)$, $x = x(t)$, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

2) $\int \frac{1}{x^n} dx = \frac{-1}{(n-1)x^{n-1}} + c$

3) $\int \frac{1}{x} dx = \log x + c$

4) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

5) $\int e^x dx = e^x + c$

6) $\int \sin x dx = -\cos x + c$

7) $\int \cos x dx = \sin x + c$

8) $\int \operatorname{cosec}^2 x dx = -\cot x + c$

9) $\int \sec^2 x dx = \tan x + c$

10) $\int \operatorname{cosec} x dx = \log [\operatorname{cosec} x - \cot x]$

11) $\int a^x dx = \frac{a^x}{\log a}, \quad a \neq 1$

12) $\int \sin ax dx = -\frac{1}{a} \cos ax$

13) $\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b)$

14) $\int \cos ax dx = \frac{1}{a} \sin ax$

15) $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$

16) $\int \tan x dx = \log [\sec x]$

17) $\int \tan ax dx = \frac{1}{a} \log [\sec ax]$

18) $\int \tan(ax+b) dx = \frac{1}{a} \log [\sec(ax+b)]$

19) $\int \tan(ax+b) dx = \frac{1}{a} \log [\sec(ax+b)]$

20) $\int \cot x dx = \log \sin x$

21) $\int \cot ax dx = \frac{1}{a} \log \sin ax$

22) $\int \cot(ax+b) dx = \frac{1}{a} \log [\sin(ax+b)]$

23) $\int \operatorname{Sec} x dx = \log [\sec x + \tan x]$

24) $\int \sec ax dx = \frac{1}{a} \log [\sec ax + \tan ax]$

25) $\int \sec(ax+b) dx = \frac{1}{a} \log [\sec(ax+b) + \tan(ax+b)]$

26) $\int \operatorname{cosec}^2 x dx = -\cot x$

Scanned with CamScanner

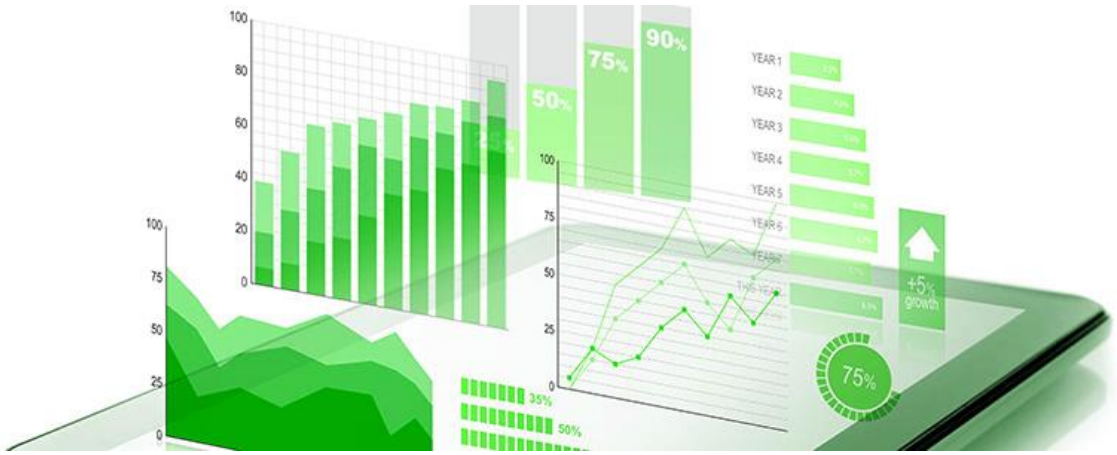
TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – IX

STATISTICS



COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

UNIT-IX STATISTICS-I

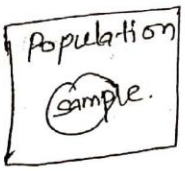
Statistics may be defined as the science of collection, presentation, analysis of data and interpretation of numerical data.

Population:

A statistical population is the set of all possible measurement on data corresponding to the entire collection of units for which an inference is to be made

Sample:

A sample is a part of the statistical population. It is a subset which is collected to draw an inference about the population



Random Sample:

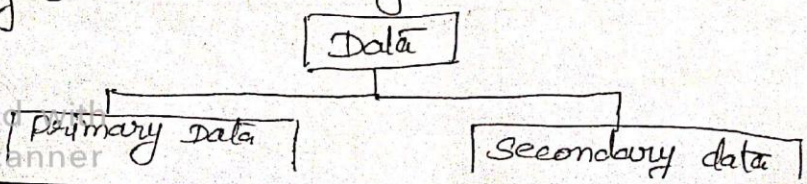
A random sample is the one where each item of the population has an equal chance of being included in the sample.

Data:

The information collected through 'census and survey or in a routine manner or other source is called a raw data. The word data means information.

Collection of data:

Collection of data is the process of enumeration together with the proper recording of results. The success of an enquiry is based upon the proper collection of data. collection of data is the first step in any statistical investigation.



Scanned with CamScanner

Primary data:

Data which are collected for the first time a they are original in character.

Secondary data:

Secondary data are those which are already collected by some one for some purpose and are available for the present study.

Classification of Data:

The process of grouping a large number of individual facts or observations on the basis of similarity among the items is called classification.

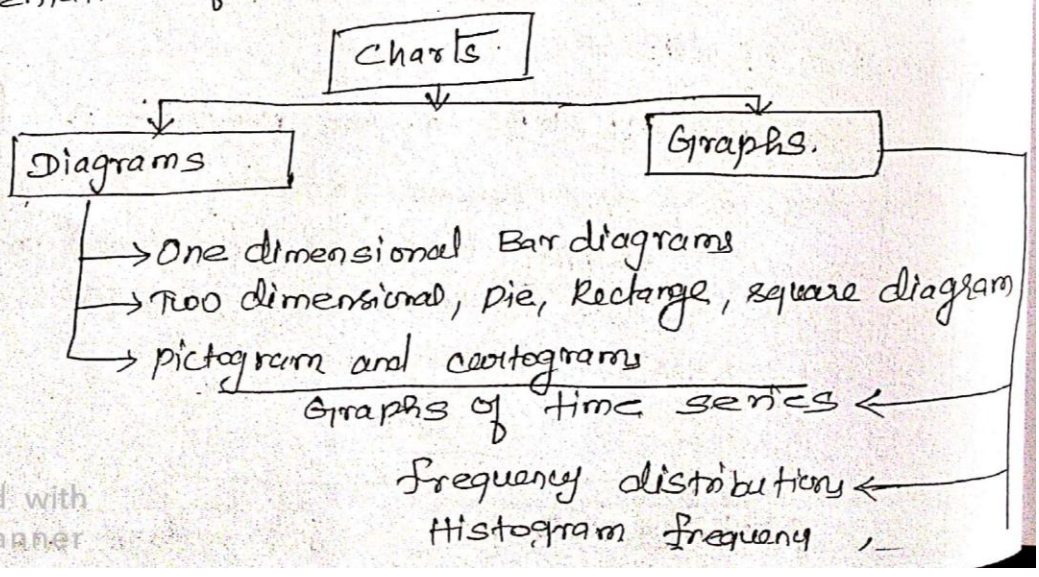
Types of classification

a) Geographical or Spatial classification
b) Chronological classification
c) Qualitative classification < simple, manifold.
d) Quantitative classification

Tabulation:

Tabulation mean a systematic presentation of numerical data in columns and rows in accordan with some salient features or characteristic

Presentation of data:



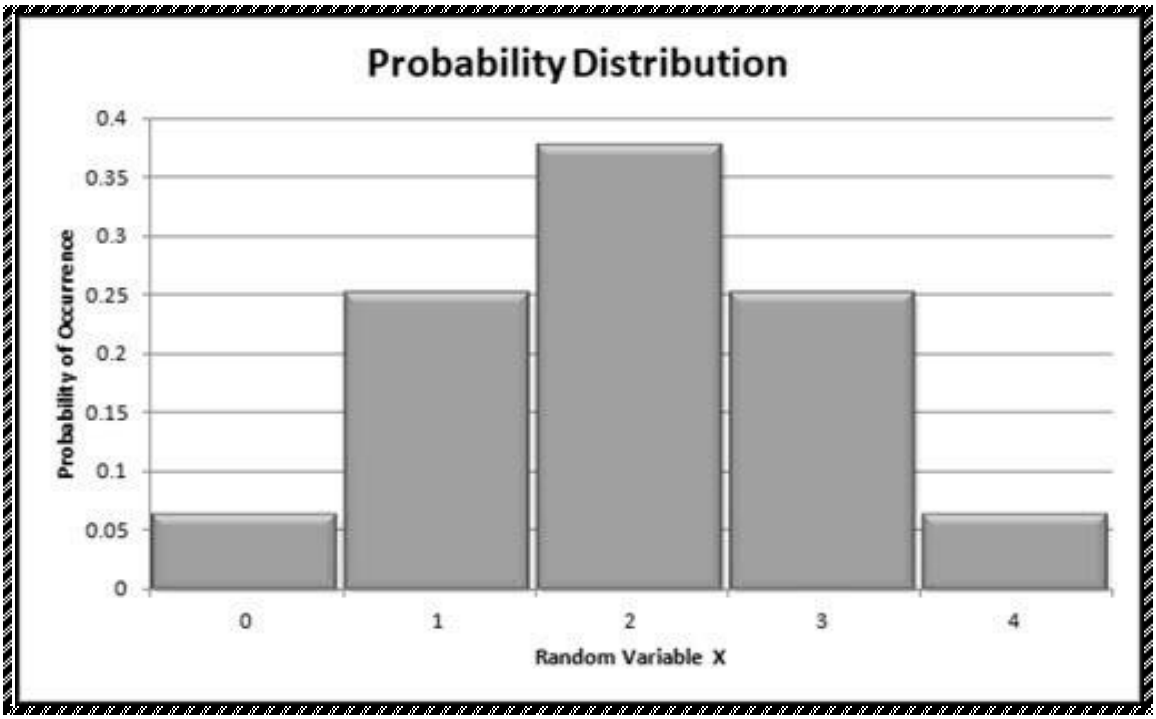
Scanned with CamScanner

TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

Unit – X - STATISTICS II



COMPETITIVE EXAM

FOR

PG-TRB 2019 – 20

7) Measure of Skewness.

$$\beta_1 = \frac{1}{\lambda} = \frac{\mu_3^2}{\mu_2^3}, \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1}{\lambda} + 3.$$

$$\gamma_1 = \frac{1}{\sqrt{\lambda}}, \qquad \gamma_2 = \frac{1}{\lambda}.$$

8) Mode:

i) If λ is not an integer then mode = λ — unimodal

ii) If λ is an integer, then mode = $\lambda - 1$ and λ — (Bimodal)

9) Recurrence relation for the moments

$$\mu_{r+1} = r\lambda \mu_{r-1} + \frac{\lambda \mu_r}{\partial \lambda}.$$

10) Recurrence relation for probabilities of poisson distribution $P(x+1) = \frac{\lambda}{x+1} P(x)$.

11) Poisson distribution is always a skewed.

12) If $X_1 \sim P(\lambda_1)$ and $X_2 \sim P(\lambda_2)$ then

$X_1 + X_2 \sim P(\lambda_1 + \lambda_2)$ $X_1 - X_2$ is not a Passion variate.

13) In general sum of independent poisson variates is again a poisson variate

If $X_i \sim P(\lambda_i)$ $i = 1, 2, \ldots, k$

$$\sum_{i=1}^{k} X_i \sim P\left(\sum_{i=1}^{k} \lambda_i\right)$$

13) Mean Deviation about mean $= \frac{2\mu \cdot e^{-\lambda} \cdot \lambda^{\mu}}{\mu!}$

Where μ is the greatest Integer contained in λ.

14) $P(x=n) = P(x=n+1) \Rightarrow \lambda = n+1$.

Scanned with CamScanner

6) The area under the normal curve within its range $-\infty$ to ∞ is always unity.

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx = 1. \quad , \quad \int_{-\infty}^{\infty} f(x)dx = 1$$

7) Mean = Median = Mode. (Normal curve is symmetrical)
The normal curve is unimodal.

8) All odd order moments of the normal distribution with zero.

9) The first raw moments ie, mean $= \mu_1$ and
$\mu_2 = \sigma^2$, $\mu_3 = 0$, $\mu_4 = 3\sigma^4$.

10) Measure of Skewness and Kurtosis

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0. \qquad \gamma_1 = \sqrt{\beta_1} = 0$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3. \qquad \gamma_2 = \beta_2 - 3 = 0$$

The value of β_1 and β_2 clearly reveals that the normal curve is symmetrical and mesokurtic.

11) Quartile Deviation $Q.D = \frac{Q_3 - Q_1}{2} = \frac{2}{3}$

12) Mean Deviation $M.D = \frac{4}{5}$

13) $M.g.f = M_{X(t)} = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2\sigma^2}(x-\mu)^2} dx = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

14) Characteristic mean

$$\phi_x(t) = \int_{-\infty}^{\infty} e^{itx} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}.$$

15) If X and Y are two normal variates such that $X \sim N(0,1)$ and $Y \sim N(0,1)$, the variate X/Y follows Cauchy distribution.

CS Scanned with CamScanner