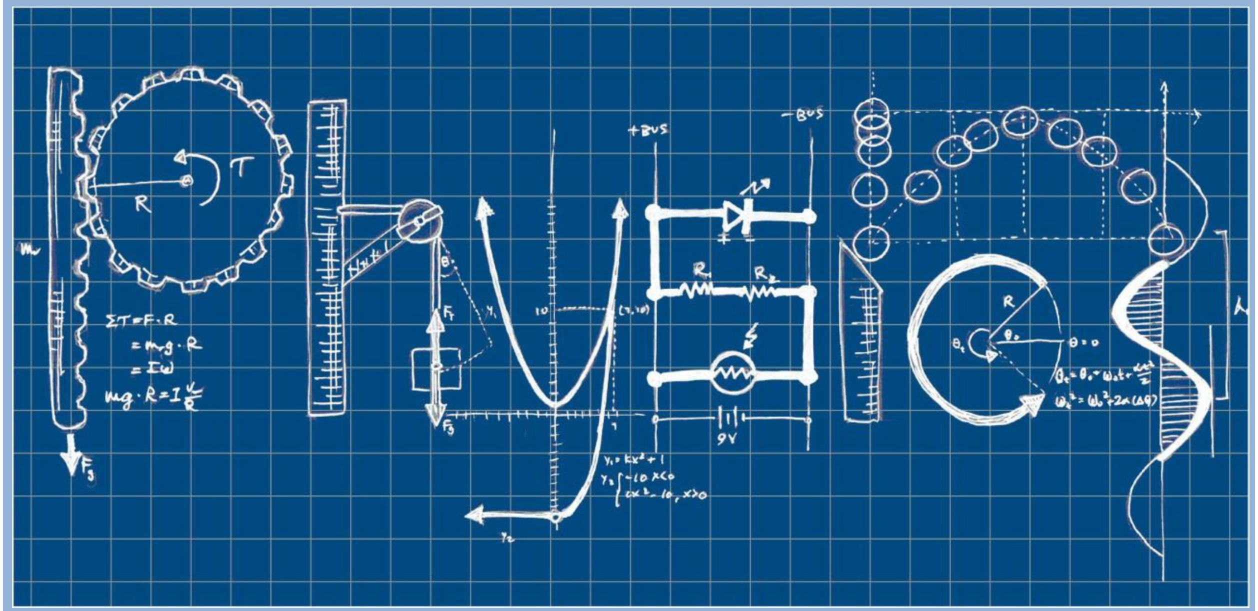
TEACHER'S CARE ACADEMY
KANCHIPURAM

PHYSICS
(UNIT -I)
Vector Fields



COMPETITIVE EXAM
FOR
PG TRB 2019-20

MATHEMATICAL PHYSICS

UNIT I — VECTOR FIELDS

The Gradient of a Scalar Field

Del operator or **Nabla operator**

$$\nabla = \vec{i}\cdot \partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z$$

- The gradient of any scalar function ϕ is defined as

$$\text{grad}\,\phi = \nabla\phi = \vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}$$
$$= [\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z]\phi = \nabla\phi$$

SOME Important formulas.

NOTE: The gradient of any scalar quantity is a vector.

1. $\nabla(\vec{a}\cdot\vec{r}) = \vec{a} = (\bar{a}\cdot\nabla)\vec{r}$
2. $\nabla r^n = \text{grad}\, r^n = \boxed{n r^{n-2}\,\vec{r}} = n r^{n-2}\vec{r}$
3. $\nabla\phi\cdot d\vec{r} = d\phi$
4. $\nabla\phi = \hat{r}$
5. $\nabla(u+v) = \nabla u + \nabla v$
6. $\nabla(u\cdot v) = u\nabla v + v\nabla u$
7. $\nabla(u/v) = \dfrac{v\nabla u - u\nabla v}{v^2}$ $\quad v \neq 0.$
8. Angle between two surfaces ϕ_1 & ϕ_2: $\cos\theta = \dfrac{(\nabla\phi_1)\cdot(\nabla\phi_2)}{|\nabla\phi_1\cdot\nabla\phi_2|}$
9. $\nabla(1/r) = \dfrac{-\vec{r}}{r^3}$
   when $\vec{r}$ = position vector of a point; $r = (x^2+y^2+z^2)^{1/2}$

Problems:

① If $\phi = x^3y^2z$ find $\nabla\phi$ at $(1,1,1)$ & find $|\nabla\phi|$

$$\nabla\phi = \vec{i}\frac{\partial(x^3y^2z)}{\partial x} + \vec{j}\,\partial/\partial y\,(x^3y^2z) + \vec{k}\,\partial/\partial z\,(x^3y^2z)$$
$$= \vec{i}\,(3x^2y^2z) + \vec{j}\,(x^3\cdot 2y\cdot z) + \vec{k}\,(x^3y^2\cdot 1)$$

$$\nabla\phi(1,1,1) = \vec{i}\,(3) + \vec{j}\,(2) + \vec{k}\,(1)$$
$$= 3\vec{i} + 2\vec{j} + \vec{k}$$

$$|\nabla\phi| = \sqrt{9+4+1} = \sqrt{14}$$ // Ans.

(2) Find $\nabla\phi$ and $|\nabla\phi|$ for the fn $\phi = 2xz^4 - x^2y$ at $(2,2,-1)$

Ans $\nabla\phi = \vec{i}\frac{\partial}{\partial x}(2xz^4 - x^2y) + \vec{j}\cdot\frac{\partial}{\partial y}(2xz^4 - x^2y) + \vec{k}\frac{\partial}{\partial z}(2xz^4 - x^2y)$

$= \vec{i}(2z^4 - 2xy) + \vec{j}(-x^2) + \vec{k}(8xz^3)$

$\nabla\phi(2,2,-1) = \vec{i}(2-8) + \vec{j}(-4) + \vec{k}(-16)$

$= -6\vec{i} - 4\vec{j} - 16\vec{k}$

$|\nabla\phi| = \sqrt{36 + 16 + 256} = \sqrt{372} = \sqrt{4\times 93} = 2\sqrt{93}$

$= \sqrt{308}$

$256 + 16 + 36 = 308$

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(3) Find the unit vector perpendicular to the surface $x^2+y^2-z^2 = 11$ at the point $(4,2,3)$

Solution: Level surface $\phi = x^2+y^2-z^2 = \text{constant}$.

Note: $\nabla\phi$ is $\perp$ to the level surface

$\therefore \nabla\phi = \vec{i}\frac{\partial}{\partial x}(x^2+y^2-z^2) + \vec{j}\frac{\partial}{\partial y}(x^2+y^2-z^2) + \vec{k}\frac{\partial}{\partial z}(x^2+y^2-z^2)$

$= \vec{i}(2x) + \vec{j}(2y) + \vec{k}(-2z)$

$= 2(x\vec{i} + y\vec{j} - z\vec{k})$ $\therefore \nabla\phi(4,2,3) = 2(4\vec{i}+2\vec{j}-3\vec{k})$

$|\nabla\phi| = 2\sqrt{16+4+9} = 2\sqrt{29}$

Formula: Unit vector $= \frac{\nabla\phi}{|\nabla\phi|} = \frac{2(4\vec{i}+2\vec{j}-3\vec{k})}{2\sqrt{29}}$

Ans $= \frac{4\vec{i}+2\vec{j}-3\vec{k}}{\sqrt{29}}$

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(4) Prove that $\nabla r^n = n r^{n-2}\vec{r}$

LHS $\nabla r^n = \vec{i}\frac{\partial r^n}{\partial x} + \vec{j}\frac{\partial r^n}{\partial y} + \vec{k}\cdot\frac{\partial r^n}{\partial z}$

$= \vec{i}\, n r^{n-1}\cdot\frac{\partial r}{\partial x} + \vec{j}\cdot n r^{n-1}\cdot\frac{\partial r}{\partial y} + \vec{k}\cdot n r^{n-1}\cdot\frac{\partial r}{\partial z}$

$= n r^{n-1}\left[\vec{i}\frac{\partial r}{\partial x} + \vec{j}\frac{\partial r}{\partial y} + \vec{k}\frac{\partial r}{\partial z}\right]$

We know that $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $\Rightarrow r^2 = x^2+y^2+z^2$

$r = (x^2+y^2+z^2)^{1/2}$

$\frac{\partial r}{\partial x} = x/r$, $\frac{\partial r}{\partial y} = y/r$, $\frac{\partial r}{\partial z} = z/r$

$\therefore \nabla r^n = n\cdot r^{n-1}\left[\vec{i}\cdot x/r + \vec{j}\cdot y/r + \vec{k}\cdot z/r\right]$

$= n r^{n-1}\frac{1}{r}\left[x\vec{i} + y\vec{j} + z\vec{k}\right]$

$= n r^{n-2}\vec{r}$ $(\because x\vec{i}+y\vec{j}+z\vec{k} = \vec{r})$

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PHYSICS

(UNIT –II)

Probability and Theory of errors

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UNIT-II - PROBABILITY & THEORY OF ERRORS

STATISTICAL PROBABILITY

Statistics: "Numerical data collected systematically (or) The Science of collecting and interpreting its information."

Probability: Definition: If a Sample Space contains n outcomes and if 'm' of them are favourable to an event A, then we write $n(S) = n$ & $n(A) = m$

- The probability of the event A is denoted by $P(A)$

(ie) $P(A) = \dfrac{\text{No of outcome favourable to A}}{\text{Total No of outcomes}}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{m}{n}$$

sample size

Note: ① The above classical definition of probability is not applicable if the No. of possible outcomes is infinite.

② The probability of an event A lies between 0 and 1, both inclusive (ie) $0 \le P(A) \le 1$

③ The probability of the sure event is 1 and the probability of impossible event is zero (0) ie $P(\phi) = 0$

④ If P is the probability of happening event A (ie) $P(A) = p$ and q is not happening event of A than $P(\bar{A}) = q = 1 - p$

$\therefore P(A) + P(\bar{A}) = 1 \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - \frac{m}{n}$

$= \frac{n-m}{n}$.

⑤ $\therefore P(A) = p$ and $P(\bar{A}) = q$

$p + q = 1$ | $p \rightarrow$ happening event (success)
$q \rightarrow$ not happening event (failure)

BASIC CONCEPTS: - If there are q. No. of exhaustive mutually exclusive and equally likely cases of an event and suppose that p of them are favourable to the happenings of an event A under the given set of conditions, then the mathematical probability of the event A is defined as $P(A) = \dfrac{p \text{ (favourable)}}{q \text{ (unfavourable)}}$

$\therefore P(A) = \frac{\text{No of favourable outcomes for A, (m)}}{\text{No of exhaustive outcomes (n)}}$

$P(A) = \frac{m}{n}$

Exhaustive – Happening of an event in favour (or) against.

Mutually exclusive – The probability of two simultaneous happenings

$P(A \cup B) = P(A) + P(B)$

Equally likely: – In trial equally probable. No happening biased

* The odd in favour of the event A are m to n (or n to m against A). The probability of happening the event A is defined as $P(A) = \frac{m}{m+n}$, $P(\bar{A}) = \frac{n-m}{n}$

(or) $P(\bar{A}) = 1 - \frac{m}{n} = 1 - P(A)$

Problem: (1) A coin is tossed. Find the probability that a head is obtained

$m = n, \quad n = 1 \quad P(A) = \frac{m}{m+n} = \frac{1}{2}$

(2) The probability of drawing a white ball from a bag containing 3 white and 4 red balls.

Let $m = 3$ and $n = 4$ $\quad P(A) = \frac{P(A)}{P(S)}$

$P(A) = \frac{3}{3+4} = 3/7$

3 A bag containing 4 white and 5 black balls, a man draws 3 at random. What are the odds against these being all black.

ANS:– Total No of ways in which 3 balls can be drawn $= 9C_3$

No of ways in which 3 black balls can be drawn $= 5C_3$

$\therefore$ Required probability $= \frac{5C_3}{9C_3} = \frac{5 \times 4 \times 3/3 \times 2 \times 1}{9 \times 8 \times 7/3 \times 2 \times 1} = \frac{5}{42}$

(4) Three cards are drawn at random from a pack of 52 cards. Find the chance that they are a king, a queen, and a knave.

ANS: No of ways for 3 cards drawn from 52 cards $= 52C_3$

The ~~bags~~ pack contain 4 queen, 4 king and 4 knave. $\frac{64}{52C_3}$

$\cdots\cdots = 64 \quad P(A) = \frac{64}{52C_3}$

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PHYSICS

(UNIT -IV)

Statistical Mechanics

COMPETITIVE EXAM

FOR

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Unit-IV Statistical Mechanics

Basics:

⇒ The statistical methods are applied to physical systems containing a very large number of particles.

⇒ Types of statistics.

1) Classical statistics
   ⇒ Maxwell - Boltzmann statistics (Ex: gas molecules)
2) Quantum statistics.
   i) Bose - Einstein statistics (Bosons ⇒ zero or integral spin. Ex: photon)
   ii) Fermi - Dirac statistics (Fermions ⇒ Half integral spin. Ex: Electron).

phase space

To specify the state of gas from the molecular point of view, we require the position and momentum of each its molecules.
We must specify six quantities x, y, z, p_x, p_y, p_z for each of its molecules.
The state of the point in the space will be described by a set of six co-ordinates x, y, z, p_x, p_y, p_z. The six-dimensional (6-D) space is called phase space, and element of volume in the space is termed as a cell.
∴ Six dimensional space phase space for a single particle is called molecular phase

molecular phase-space (or) μ-space and. E

6N Dimensional phase space is called

T-space (or) γ-space.

=> The dimension of the volume element

are $(\text{length} \times \text{momentum})^{3} = (\text{Joule}\cdot\text{second}$

=> The size of the cell (each) be h.

h - constant has the dimension of joule second.

Micro states (2^n)

we must state to which cell each molecule of the system belongs temporarily.

Ex: In the 4 particles, the total number of microstates $= 2^n = 2^4 = 16$.

Macro states $(n+1)$:

The specifications of the number of molecules (or) phase points in each cell of phase-space.

Ex: In the 4 particles, The total number of macrostates $= (n+1) = (4+1) = 5$

=> many different microstates may correspond to same macrostate.

=> The microstate which are allowed under given restrictions are called accessible microstate.

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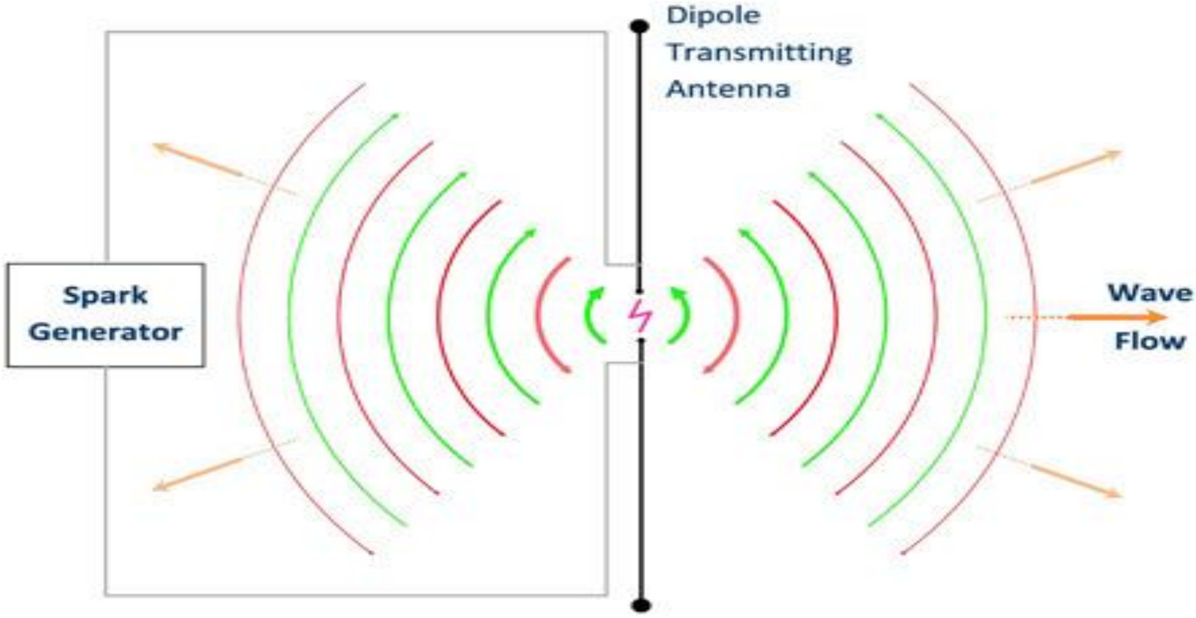
PHYSICS

(Unit – V)

Electro Magnetic Theory



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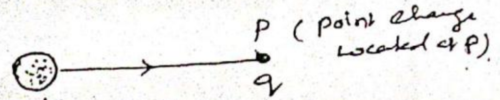
UNIT - V - ELECTROMAGNETIC THEORY

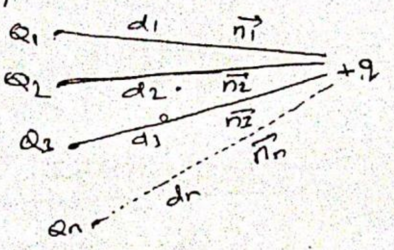
Electrostatics - study about static or stationary Electric charge.

SOME KEY TERMS:



(i) POINT CHARGE Distribution: source charge

when a charge is concentrated at a point, It is called point charge



(ii) Discrete Charge distribution

- The force of interaction between the two charges is unaffected by presence of any other charges in the vicinity.
- The Electric field intensity at a point of consideration due to another point charge is unaffected by the presence of another point charge.

$$E_1 = 9\times10^9 \frac{Q_1}{d_1^2}\hat{n_1}$$

$$E_2 = 9\times10^9 \frac{Q_2}{d_2^2}\hat{n_2}$$

$$\therefore \vec{E} = \vec{E_1} + \vec{E_2} + \dots \vec{E_n}$$

(iii) Continuous Charge distribution → Line charge distribution, → ~~Area~~ surface charge ", → Volume charge distribution.

- If the charge is continuously distributed in the form of line, surface or volume rather than at a point is called continuous charge distribution.

* Line charge distribution: when a charge is concentrated over a line with a line (or) linear charge density λ.

- Linear density of charge (λ) - one dimensional

$$\lambda = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l}$$

unit: $\frac{\text{coulomb}}{\text{metre}}$ (ie) $C m^{-1}$

Surface charge distribution: charge is distributed over a surface with surface charge density σ

- Surface charge density (σ) - 2 dimensional

$$\sigma = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s}$$

$\frac{\text{coulomb}}{m^2}$ (ie) $C m^{-2}$

Δs - is the surface with negligible thickness.

Volume charge distribution : The volume charge distribution having volume charge density ρ

- volume charge density (ρ) – Three dimensional

If Δq is the net charge enclosed by volume $\Delta\tau$ or ΔV

$$\rho = \lim_{\Delta\tau \to 0} \frac{\Delta q}{\Delta\tau}$$

$q = ne$ (elementary charge)

$n \to$ no of charged particle in unit volume.

$e \to$ charge of each particle.

Unit : coulomb/m^3

COULOMB'S LAW : STATEMENT :

"The force of attraction or repulsion between two point charges is directly propositional to the product of the charges and inversely propotional to the square of the distance between them.

q_1 ------- q_2

$$F = K\frac{q_1 q_2}{r^2} \text{ Newton.}$$

where K = constant

$$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \; Nm^2C^{-2}$$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

ε_0 – permittivity in free space = 8.854×10^{-12} Farad/m or $C^2N^{-1}m^{-2}$

- coulomb law is applicable for values of r greater than 10^{-14} m

Note : Other forms of coulomb's law

* In the presence of dielectric between two point charges

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2}$$

$\boxed{\varepsilon = \varepsilon_0\varepsilon_r.}$

$\varepsilon \to$ permittivity of medium

$\varepsilon_r \to$ Relative permittivity (Dielectric)

$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$ (No dimension)

* Vector form of coulomb's law.

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{n}$$

where F_{12} = Force exerted by q_1 on q_2

$\hat{n}$ = unit vector along the direction of force.

Note for Air medium $\boxed{\varepsilon_r = 1}$

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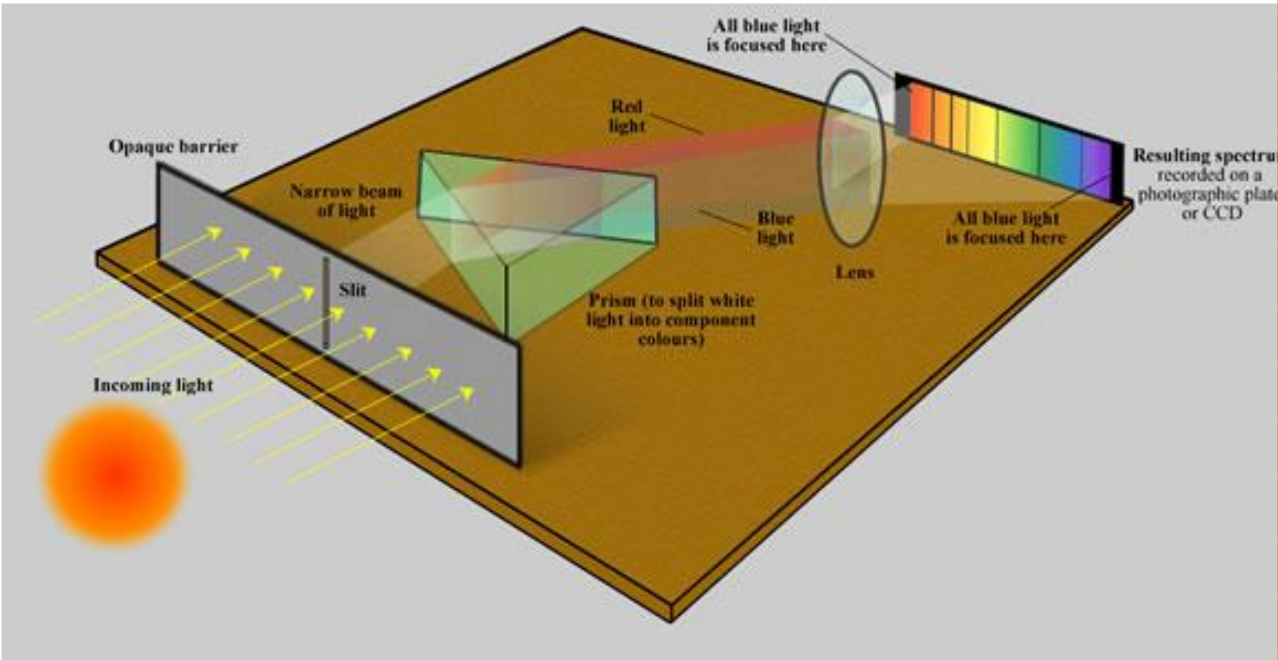
PHYSICS

(UNIT -VI)

Spectroscopy



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UNIT - VI

SPECTROSCOPY.

→ spectroscopy is the most powerful tools for study of atomic and molecular

ATOMIC SPECTROSCOPY:

→ It deals with the interaction of electromagnetic radiation with atoms. which commonly in their lowest energy state is called ground state.

MOLECULAR SPECTROSCOPY:

→ It deals with interaction of electromagnetic radiation with molecules. This results in transition between rotational, vibrational and electronic transition.

Molecular spectra:

1.) Electronic spectra

2.) Vibration-rotational spectra

3.) pure rotation spectra.

1.) Electronic spectra.

* Both in emission and absorption in visible and UV region.

* The electronic spectra observed for hetro and homo nuclear diatomic molecules ($10^{-2}\mu$ - $10^{0}\mu$).

* The energy $E = 10 eV$
* permanent dipole moment not necessary.

2.) Vibrational - rotational spectra
* These spectra are observed in absorption in the near IR region ($1\mu - 10^2\mu$).
* The energy $E = 10^{-1} eV$
* vibration rational spectra are observed onl both hetro nuclear molecule.
* Homonuclear do not produce vibrational rotational

3.) Pure rotation spectra:
* These spectra are observed in absorption in the far Infrared region ($10^2\mu - 10^3\mu$)
* Microwave region ($= 10^3\mu - 10^4\mu$)
* The energy $E = 10^{-1} eV$.
* The pure rotational spectra are observed on for the hetro nuclear diatomic molecules.
* permanent dipole moment is necessary.

The total or internal energy of the diatomic molecules may be writen as:

$$E(internal) = E(electronic) + E(vibrational) + E(rotal$$

E(internal) → internal energy may posses a molecule

$$E_{ele} > E_{vib} > E_{rot}.$$

If E_{ele} and E_{vib} constant. Then the transition between in different rotational energy level gives pure rotational spect

Rotational spectra:
→ Microwave spectroscopy is otherwise known as pure rotational spectroscopy.
→ The rotational energy of molecules and occurs in the frequency range from 3 to 300 GHz
→ spectra occur in Far IR region for light molecules ($10^2\mu$-

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PHYSICS

(UNIT –VIII)

COMPETITIVE EXAM

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16/8/15

Quantum Mechanics UNIT-VIII

i) classical Mechanics could not explain spectrum of block body radiations.

ii) It could not explain a large no. of observed phenomena like a photo electric effect compton effect, Ramman effect etc.

iii) It could not explain the variation of specifi heat of metals and gases.

iv) classical mechanics could not explain the Origin of discrite spectra of atoms.

Matter wave :-

According to de Broglie has wave propert associated with it. Idea of dual nature (wave - praticale duality)

The wave length λ associated with any moving particle of momentum p is given

by $\boxed{\lambda = \frac{h}{P} = \frac{h}{mv}}$

Here $h \rightarrow$ planks constant

According to this concepts, Only those orbits are allowed as stationary orbits whose circumference is integral multiple of wave length associated with electron

$$\boxed{2\pi r = n\lambda}$$

Here r is radius of orbit

$n = 1, 2, 3 \ldots$

single wave **Wave or phase Velocity :-**

→ When a single wave of a definite wavelength (same) travels in a medium its Velocity (same) of propagation in the medium is called the "Wave Velocity (or) phase Velocity"

Group Velocity :-

→ If a number of waves of different wavelengths are moving with different Velocities (diff) in a medium, then the observed Velocity (different) is called the Group Velocity

It is the Velocity of the wave packet (or wave group) formed by the waves.

TEACHER'S CARE ACADEMY

KANCHIPURAM

PHYSICS

(UNIT -X)

COMPETITIVE EXAM

FOR

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UNIT - X

Digital Electronics

Bit → Binary Digits

Byte → String of 8 Bits

1 kilobyte = 1kB = 1024 bytes = 2^{10} bytes.

1GiB = 1TB

Byte – combination of 8 bits

→ It is a basic unit of binary information and storage.

MSB – Most significant Bit

LSB – Least significant Bit

416 Lakhs.

MSB LSB

Number system	Base (Radix)	Allowed digits
Binary	2	0, 1
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
octal	8	0, 1, 2, 3, 4, 5, 6, 7
Hexadecimal	16	0 ... 9, A, B, C, D, E, F

1) convert $(101)_2$ into decimal.

$$(101)_2 = 1\times2^2 + 0\times2^1 + 1\times2^0$$

$$= 4+0+1$$

$$(101)_2 = (5)_{10}$$

2) convert $(1101011.1011)_2$ into decimal

1101011 . 1011

$1\times2^0 = 1$
$1\times2^1 = 2$
$0\times2^2 = 0$
$1\times2^3 = 8$
$0\times2^4 = 0$
$1\times2^5 = 32$
$= 64$

$$= 1\times2^6 + 1\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2 \times 1\times2 + 1\times2$$

$$= 64+32+0+8+0+2+1$$

$$= 107$$

$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25$

$\frac{1}{8} = 0.125$

$$1011 = 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} + 1\times2^{-4}$$

$\frac{1}{16} = 0.0625$

$$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}$$

0.0625
0.125
0.5
0.6875

$$= 0.5 + 0 + 0.125 + 0.0625$$

$$= 0.6875$$

$$(1101011.1011)_2 = (107.6875)_{10}$$