TEACHER'S CARE ACADEMY

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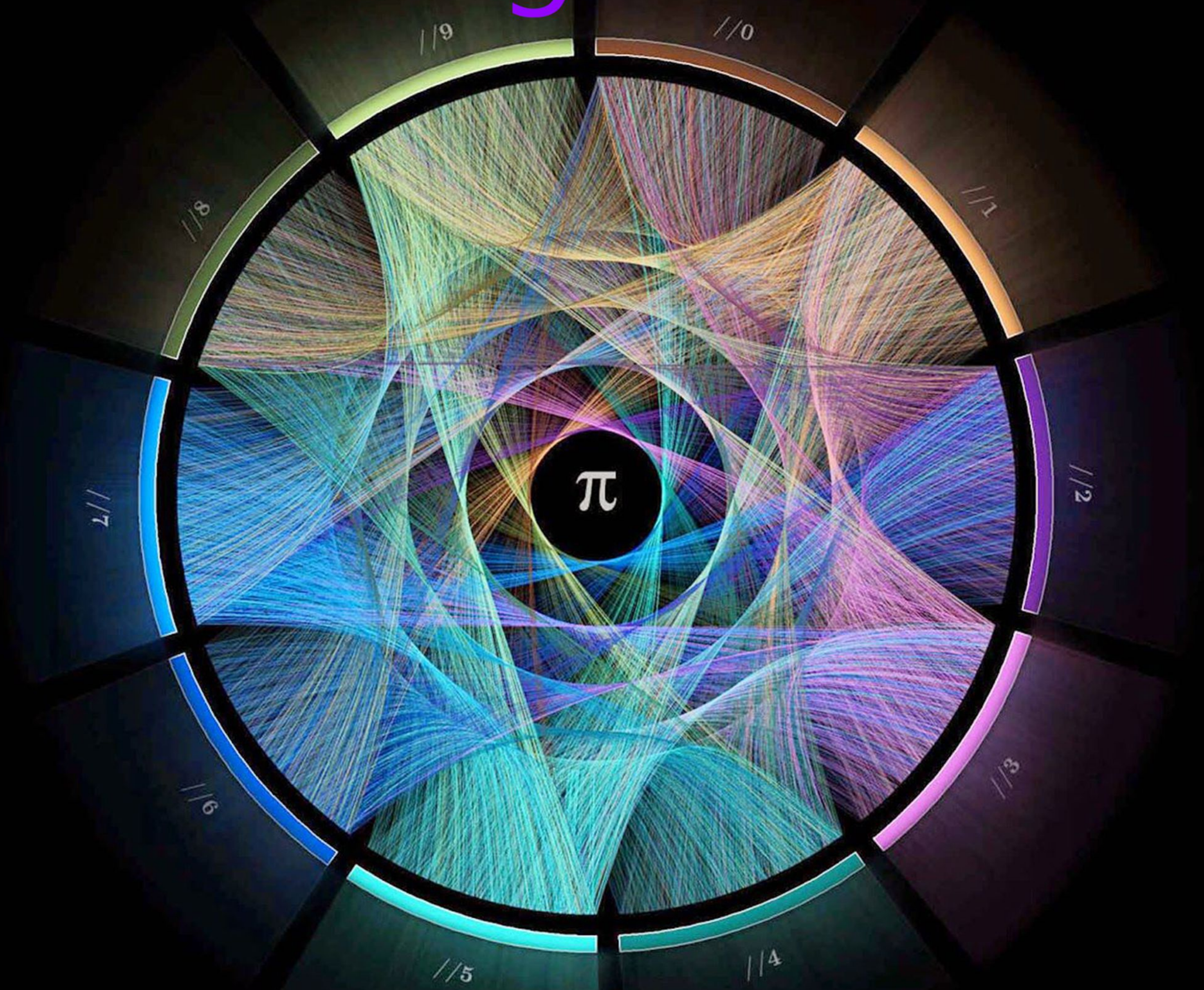
MATHEMATICS

Algebra



PG TRB

2020-2021

UNIT-1

SYLLABUS: MATHEMATICS

UNIT- I

ALGEBRA

Groups – Examples – Cyclic Groups- Permulation Groups – Lagrange's theorem- Cosets – Normal groups - Homomorphism – Theorems – Cayley's theorem - Cauchy's Theorem - Sylow's theorem - Finitely Generated Abelian Groups – Rings- Euclidian Rings- Polynomial Rings- U.F.D. - Quotient - Fields of integral domains- Ideals- Maximal ideals - Vector Spaces - Linear independence and Bases - Dual spaces - Inner product spaces - Linear transformation – rank - Characteristic roots of matrices - Cayley Hamilton Theorem - Canonical form under equivalence – Fields - Characteristics of a field - Algebraic extensions - Roots of Polynomials - Splitting fields - Simple extensions – Elements of Galois theory- Finite fields

UNIT - I.

ALGEBRA.

Groups & Some Examples

Example : This is the motivating example for the definition of Groups. Let us consider $(\mathbb{Z}, +)$

$\mathbb{Z}$ - is the set of all integers.

$\mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$

$+$: is the binary operation

(i.e) $+$ is the function from $\mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$

$+ : (a, b) \longmapsto a + b.$

This sort of function is called binary operation.

Observation in this example $(\mathbb{Z}, +)$

(i) : Let a, b be any two integers. Then $a+b$ is clearly an integer.

This prop. is called closure. (CLOSURE).

If the set has only this property The set is called Megma.

(ii) Let a, b, c be three integers then $a+b+c = ?$

We know how to operate 2 integers but what is the defn that we can define on $a+b+c$

This property is called associativity (ASSOCIATIVITY)

1. $a+b+c \overset{\text{defn}}{=} (a+b)+c \longrightarrow$ ①

(or)

2. $a+b+c \overset{defn}{=} a+(b+c)$ ⟶ ②

If the set has upto this property the set is called (semi group).

If ① ≠ ② then we cannot able to define addition on 3 elements.

But in integer

$(a+b)+c = a+(b+c)$

So we have a well-defined addition operation on 3 elements.

(iii) : There is a magic number in $\mathbb{Z}$ say 0.

$\underline{\underline{0}} + a = a + \underline{\underline{0}} = a$

This property is called Identity. (Existence of IDENTITY)

The set has upto this property then the set is called (MONOID)

(iv) and for all element

Eg : $5 \in \mathbb{Z}$ then is -5

s.t $5 + (-5) = 0$.

or for -8 ∃ $8 \in \mathbb{Z}$

s.t $-8+8 = 0$.

This property is called (INVERSE)

The set has upto this property the set is called (GROUP).

(v) In integer

$3+7 = 7+3 = 10$.

In general

$a+b = b+a$

This property is called (Commutative)

The set has this property is called (Abelian group).

Define (Group)

A non-empty set G together with an binary operation '$\star$' (i.e) $\star : G \times G \rightarrow G$) is said to be a group.

If (i) $(G, \star)$ satisfies closure.

(i.e) $\forall\, a, b \in G \Rightarrow a \star b \in G$.

(ii) $(G, \star)$ satisfies Associative.

(i.e) $\forall\, a, b, c \in G$.

$(a \star b) \star c = a \star (b \star c)$

(iii) $\exists\, e \in G$ such that

$a \star e = e \star a = a$.

e is called Identity element.

(iv) for every $a \in G$ $\exists\, a^{-1} \in G$.

Such that $a \star a^{-1} = a^{-1} \star a = e$

If the set G with the binary operation $\star$ has the above properties then $(G, \star)$ is called Group (or) we can say G is a Group under $\star$.

In addition to the First four properties

(v) $\forall\, a, b \in G$ $a \star b = b \star c$ (Commutative law)

Then $(G, \star)$ is called Abelian Group.

①. Example :

(i) prove that $(\mathbb{Z}, +)$ is an abelian group.

Proof (1) for any integer a & b

$a + b$ is also an integer.

① Let G be a group. Let $a \in G$ and $O(a) = n$ then $O(a^{-1})$ is

(a) n^2 (b) $n-1$ (c) $n - O(G)$ (d) n.

② Let G be a group $a, b \in G$ $O(a) = n$ $O(b) = m$ then $O(ab)$ is

(a) mn (b) $gcd(m,n)$ (c) $Lcm(m,n)$ (d) n.

③ Let G be a group, If a and b are two elements of order 8 and 10 respectively, then the order of element $a^{-1}b$ is

(a) 80 (b) 18 (c) 2 (d) 40

④ Let $\mathbb{C}^*$ be the group $\mathbb{C}/\{0\}$. Then a finite subgroup of $\mathbb{C}^*$.

(a) is contained in $\mathbb{R}^*$ (b) consist of only 1 & -1.

(c) is contained in $\mathbb{Q}^*$ (d) is contained in $\{z \in \mathbb{C} : |z| = 1\}$

⑤ If G is a group of order 20, then the number of subgroups of G of order 5 is

(a) 1 (b) 4 (c) 5 (d) 2.

⑥ If the order of every non-trivial element in a group is n, then

(a) n is necessarily a prime number.

(b) n can be any odd number

(c) n is an even number.

(d) n can be +ve integer.

(7) Let G_1 & G_2 be two finite group of prime order $\phi : G_1 \to G_2$ be an homomorphism then

(a) ϕ is necessarily trivial (b) ϕ is necessarily one to one.

(c) ϕ is necessarily onto (d) none of the above.

(8) Which of the following could be an order of a non-abelian group?

(a) 4 (b) 8 (c) 9 (d) 13.

(9) The number of subgroups of the cyclic group G of order 15, excluding the trivial group and G is

(a) 2 (b) 3 (c) 13 (d) 14.

(10) Let S_4 be the group of permutation on four letters. The number of elements of order 2 in the group S_4 is

(a) 6 (b) 9 (c) 4 (d) 12.

(11) The number of abelian groups of order 27 is

(a) 1 (b) 2 (c) 3 (d) 12.

(12) Let G be a group of order 4 then G is

(a) cyclic (b) abelian (c) permutation group

(d) none of the above.

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Mathematics

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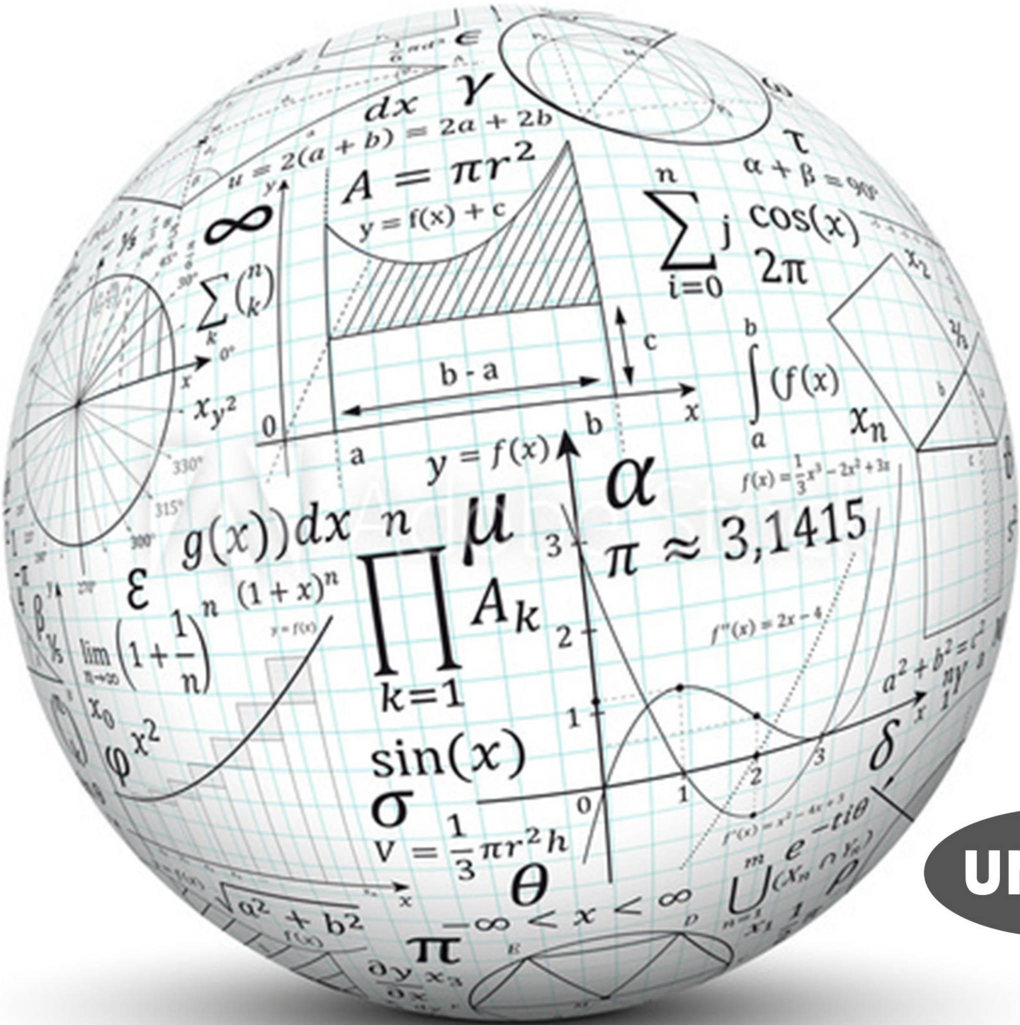

UNIT-2

Real Analysis

SYLLABUS: MATHEMATICS

UNIT- II

REAL ANALYSIS

Cardinal numbers - Countable and uncountable cordinals - Cantor’s diagonal process – Properties of real numbers - Order - Completeness of R-Lub property in R-Cauchy sequence - Maximum and minimum limits of sequences - Topology of R.Heine Borel - Bolzano Weierstrass - Compact if and only if closed and bounded - Connected subset of R-Lindelof’s covering theorem - Continuous functions in relation to compact subsets and connected subsets- Uniformly continuous function – Derivatives – Left and right derivatives - Mean value theorem - Rolle’s theorem- Taylor’s theorem- L’ Hospital’s Rule - Riemann integral - Fundamental theorem of Calculus – Lebesgue measure and Lebesque integral on R’Lchesque integral of Bounded Measurable function - other sets of finite measure - Comparison of Riemann and Lebesque integrals - Monotone convergence theorem - Repeated integrals.

REAL ANALYSIS.

Defn (function):

Let A & B be two sets and let f be a mapping of A into B. If $E \subset A$, $f(E)$ is defined to be the set of elements $f(x)$, for $x \in E$. We call $f(E)$ the image of E under f. $f(A)$ is the range of f. It is clear that $f(A) \subset B$.

(*) ⊙ If $f(A) = B$, we say that f maps A onto B.

⊙ If $E \subset B$, $f^{-1}(E)$ denotes the set of all $x \in A$ such that $f(x) \in E$. We call $f^{-1}(E)$ the Inverse image of E under f.

Let $E \subseteq B$; $f^{-1}(E) = \{x \in A \mid f(x) \in E\}$

Let $y \in B$ $\quad f^{-1}(y) = \{x \in A \mid f(x) = y\}$.

(*) ⊙ We say function f from A to B 1-1 iff $f^{-1}(y)$ consist of atmost one element for each $y \in B$.

In other words we say f is 1-1 iff

$f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ $\forall x_1, x_2 \in A$.

Defn: If there exist 1-1 mapping of A onto B, we say that A & B can put 1-1 correspondence, or that A & B have the same cardinal numbers, or briefly.

A & B are equivalent, and we write $A \sim B$.

'$\sim$' is an equivalence relation.

(i.e) '$\sim$' is reflexive $A \sim A$.

'$\sim$' is symmetric if $A \sim B$ then $B \sim A$.

'$\sim$' is Transitive if $A \sim B$ & $B \sim C$ then $A \sim C$.

$J_n - \{1, 2, \cdots, n\}$ $J \sim$ set of all Natural numbers.

Defn:

(i) A is finite If $A \sim J_n$ for some n.

(ii) A is Infinite If A is NOT finite.

(iii) A is countable If $A \sim J$.

(iv) A is uncountable If A is neither finite nor countable.

(v) A is atmost countable if A is finite or countable.

(Countable sets are sometimes called enumerable or denumerable sets).

Remark: set of all Integers is countable.

let $f : J \to \mathbb{Z}$ defined by

$$f(n) = \begin{cases} n/2 & (n \text{ is even}) \\ -\left(\frac{n-1}{2}\right) & (n \text{ is odd}). \end{cases}$$

So we have mapped all even natural numbers to the integers and odd natural numbers to -ve integers and 0 to 0.

Test to check set is Infinite :

(I) (A finite set cannot be equivalent to its proper subset).

"A" is infinite if A is equivalent to one of its proper subsets.

Example : By previous remark we have proved $J \sim \mathbb{Z}$ but we all know that J is a proper subset of $\mathbb{Z}$.

(II) ⊙ Let X be a countable Infinite set then we write $\boxed{|X| = \aleph_0}$ (Alaph not)

If X is finite say n then $|X| = n$.

(III) ⊙ Every countable Infinite set is $\sim$ to a proper subset of itself (proof by Axiom of choice).

(IV) ⊙ Every Infinite set has a countable infinite subset consequently.

$$A \text{ is inf set } \Rightarrow |A| \geq \aleph_0.$$

(V) ⊙ Every Infinite set is equivalent to one of its proper subsets.

Proofs ofor (I), (II), ... (V)

(I) Let X be countable infinite subset.

① Let $f(x) = \frac{1}{(x^2-6x+8)}$ What is $\lim_{x\to 2} f(x)$

ⓐ ∞ ⓑ $-\infty$ ⓒ does NOT exist ⓓ 0.

② Let $f(x) = \frac{1}{x^2-6x+8}$ What are all the set of discontinuity

ⓐ $\{2,4\}$ ⓑ $\{0,2,4\}$ ⓒ $\{2,8\}$ ⓓ $\{4,8\}$

③ Let $f(x) = \frac{1}{x^2-6x+8}$ the point 2 is what discontinuity?

ⓐ type I (essential) (b) type II (Jump) (c) Removable (d) It is cts at 2.

④ Let $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0. \\ -3 & \text{if } x = 0. \end{cases}$

then the point $x=0$ is what discontinuity.

(a) type I (essential (b) type II (Jump) (c) Removable (d) It is cts at 2.

⑤. Let $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0. \\ -3 & \text{if } x = 0 \end{cases}$ evaluate $\lim_{x\to 0^-} f(x)$

(a) -3 (b) 0 (c) 2 (d) does NOT exist.

⑥ $f(x) = \frac{|x|}{x}$ (i.e) $f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then 0 is what discontinuity?

(a) Jump (b) essential (c) Removable (d) It is cts at 0.

⑦ $\left(\lim_{x\to 0} \sin 1/x, \lim_{n\to\infty} \sin(1/n)\right) = (x,y)$ then $(x,y)=$

(a) $(0,0)$ (b) (does NOT exist, 0) (c) (does NOT exist, does NOT exist) (d) $(1,0)$

(8). What is the value of $\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)$ (use L-Hospital rule)

(a) $\sqrt{2}$ (b) $\log\sqrt{2}$ (c) $\log(2/3)$ (d) $\log 2$.

(9). Let $f:\mathbb{R}\to\mathbb{R}$ be a cts and 1-1 fn, Then WOTF is true?

(1) f is ONTO (2) f is either strictly decr. or incr.

(3) $\exists!\, x\in\mathbb{R}$ st. $f(x)=1$ (4) f is unbounded.

(10). Consider a function $f:\mathbb{R}\to\mathbb{R}$. Then WOTF are true? (Multiple select Question)

(1) f is not one-one if the graph of f intersects some line parallel to x-axis in atleast two points.

(2) f is one-one if the graph of f intersects any line parallel to the x-axis in atmost one point.

(3) f is surjective if the graph of f intersects every line parallel to x-axis.

(4). f is NOT surjective if the graph of f doesnot intercept atmost one line parallel to x-axis.

(11). f is monotone increasing function then what is the possible discontinuity set of f.

(a) $\mathbb{N}$ (b) $\mathbb{R}/\mathbb{Q}$ (c) $[0,1]$ (d) $(0,\infty)$

(12). $f:(0,\infty)\to\mathbb{R}$ be uniform continuous then

(a) $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to\infty} f(x)$ exist.

(b) $\lim_{x\to 0^+} f(x)$ exist but $\lim_{x\to\infty} f(x)$ need not exist

(c) $\lim_{x\to 0^+} f(x)$ need not exist but $\lim_{x\to\infty} f(x)$ exist

(d) None of the limits exist.

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Mathematics

PG TRB (2020-2021)

Fourier series & Fourier Integrals

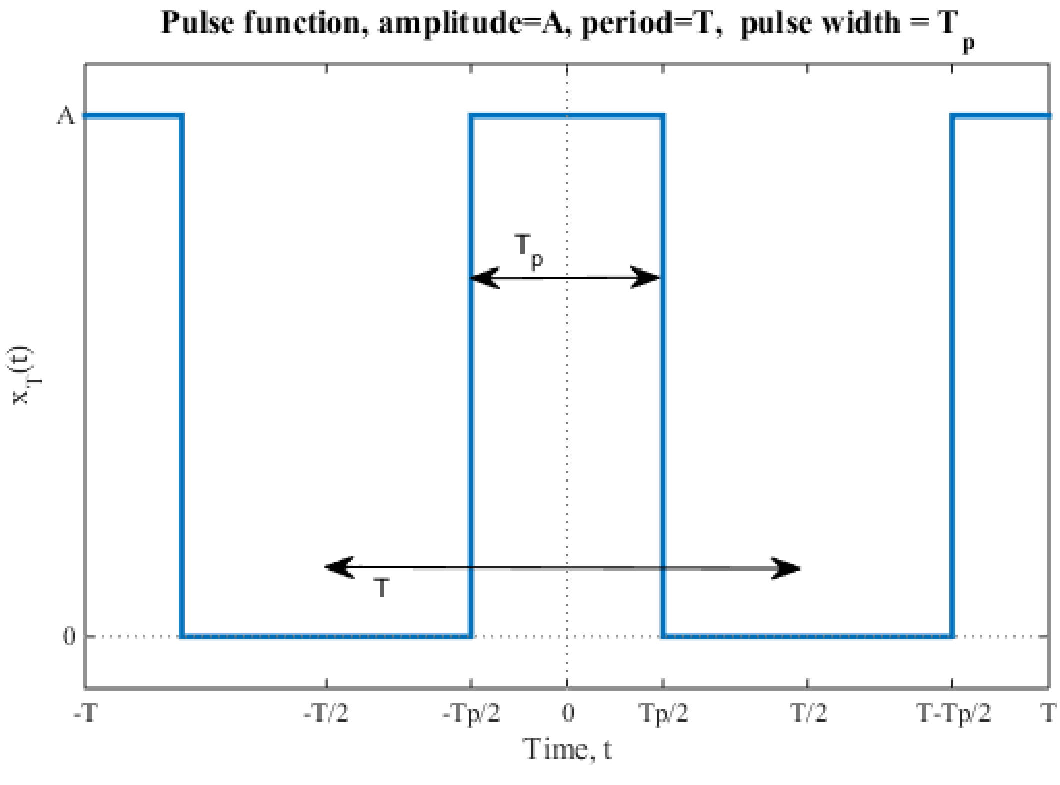
UNIT-3

TEACHER'S CARE ACADEMY

KANCHIPURAM

MATHEMATICS

UNIT – III (Vol 1)



COMPETITIVE EXAM

FOR

PG-TRB 2020 – 21

SYLLABUS: MATHEMATICS

UNIT- III

FOURIER SERIES AND FOURIER INTEGRALS

Integration of Fourier series - Fejer's theorem on (C.1) summability at a point - Fejer's-Lebsque theorem on (C.1) summability almost everywhere – Riesz-Fisher theorem - Bessel's inequality and Parseval's theorem - Properties of Fourier co-efficients - Fourier transform in L (-D, D) - Fourier Integral theorem - Convolution theorem for Fourier transforms and Poisson summation formula.

Fourier Series.

1. Consider the following trigonometric series

$$\frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots$$

Where a's and b's are constants and x a variable.

Every term except the first term has a period of 2π and consequently any function represented by a series of the above form in an interval of length 2π will also be periodic with period 2π. If the series converges in any closed interval, say $\lambda \le x < \lambda + 2\pi$, then the series is convergent for every real value of x since the series represented by the function is periodic.

2. Suppose that a given function f(x) can be expressed as a trigonometric series as

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots \rightarrow (1).$$

Let us assume that the series is uniformly convergent in the interval $\lambda \le x \le \lambda + 2\pi$.

Then the series can be integrated term by term.

To determine the a's and b's in the series, the following identities have to be used :

(i) $\int_{\lambda}^{\lambda+2\pi} \cos nx \, dx = 0$ where n is an integer.

(ii) $\int_{\lambda}^{\lambda+2\pi} \sin nx \, dx = 0$ where n is an integer.

(iii) $\int_{\lambda}^{\lambda+2\pi} \cos mx \cos nx \, dx = 0$ if $m \neq n$ and m and n are integers.

(iv) $\int_{\lambda}^{\lambda+2\pi} \sin mx \sin nx \, dx = 0$ if $m \neq n$ and m and n are integers.

(v) If $m = n$ and m and n are integers, then

$$\int_{\lambda}^{\lambda+2\pi} \cos mx \cos nx \, dx = \int_{\lambda}^{\lambda+2\pi} \cos^2 mx \, dx = \pi.$$

$$\int_{\lambda}^{\lambda+2\pi} \sin mx \sin nx \, dx = \int_{\lambda}^{\lambda+2\pi} \sin^2 mx \, dx = \pi.$$

$$\int_{\lambda}^{\lambda+2\pi} \sin mx \cos nx \, dx = \frac{1}{2}\int_{\lambda}^{\lambda+2\pi} \sin(2mx) dx = 0.$$

If we integrate both sides of the equation (1), we have

$$\int_{\lambda}^{\lambda+2\pi} f(x) dx = \int_{\lambda}^{\lambda+2\pi} \frac{a_0}{2} dx = \pi a_0.$$

$$\therefore a_0 = \frac{1}{\pi}\int_{\lambda}^{\lambda+2\pi} f(x) dx \rightarrow (2).$$

If both sides of the equation (1) are multiplied by $\cos nx$ and integrating term by term from λ to $\lambda+2\pi$, we see that all the terms on the right side vanish except the term containing a_n.

$$\therefore \text{We have } \int_{\lambda}^{\lambda+2\pi} f(x) \cos nx \, dx = a_n \pi$$

$$\therefore a_n = \frac{1}{\pi}\int_{\lambda}^{\lambda+2\pi} f(x) \cos nx \, dx \rightarrow (3).$$

Similarly, multiplying both sides of the equation (1) by $\sin nx$ and integrating we have

$$b_n = \frac{1}{\pi}\int_{\lambda}^{\lambda+2\pi} f(x) \sin x \, dx \rightarrow (4)$$

In (3), if $n = 0$ is substituted, a_0 is obtained.

Hence we have the result that if $f(x)$ can be expressed as a trigonometric series of the form.

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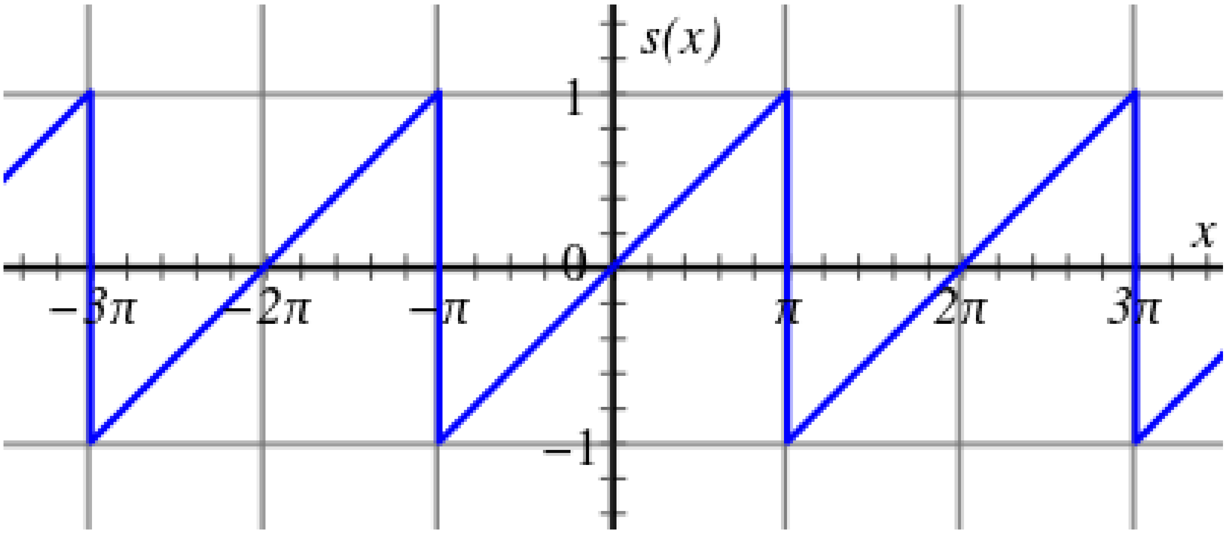
MATHEMATICS

UNIT – III (Vol 2)



COMPETITIVE EXAM

FOR

PG-TRB 2020 – 21

PG TRB 2020-21 MATHEMATICS

Unit - III (VOL-2)

Fourier Series and Fourier Integrals.

It has been shown by Fourier that a function f(x) which has only a finite number of discontinuities can be expressed as a trigonometric series in a given range of x in the form

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots + a_n \cos nx + \dots \infty)$$
$$+ (b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots + b_n \sin nx + \dots \infty)$$

Fourier has shown that the expansion of f(x) in the above form is possible only if it satisfies certain conditions. These conditions called Dirichlet conditions are stated below.

Let f(x) be defined in the interval $c < x < c+2\pi$ with period 2π and satisfy the following conditions.

i) f(x) is single valued.

ii) It has a finite number of discontinuities in a period of 2π.

iii) It has a finite number of maxima and minima in a given period.

iv) $\int_c^{c+2\pi} |f(x)|\, dx$ is convergent

If f(x) satisfies the above conditions then it is possible to express f(x) as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{—— (1)}$$

This representation of f(x) is called a Fourier expansion or a Fourier Series. Here the coefficients a_0, a_n, b_n are called Fourier coefficients. Before determining them, we state the following properties of definite integral which are used in the evaluation of a_0, a_n and b_n.

1) $$\int_c^{c+2\pi} \sin nx \, dx = \left[-\frac{\cos nx}{n}\right]_c^{c+2\pi} = 0.$$

2) $$\int_c^{c+2\pi} \cos nx \, dx = \left[\frac{\sin nx}{n}\right]_c^{c+2\pi} = 0.$$

3) $$\int_c^{c+2\pi} \cos mx \cos nx \, dx = \frac{1}{2}\int_c^{c+2\pi} [\cos(m+n)x + \cos(m-n)x] \, dx$$

$= 0$ if $m \neq n$.

$= \pi$ if $m = n$.

4) $$\int_c^{c+2\pi} \sin mx \sin nx \, dx = \frac{1}{2}\int_c^{c+2\pi} [\cos(m-n)x - \cos(m+n)x] \, dx$$

$= 0$ if $m \neq n$

$= \pi$ if $m = n$.

5) $$\int_c^{c+2\pi} \sin mx \cos nx \, dx = \frac{1}{2}\int_c^{c+2\pi} [\sin(m+n)x - \sin(m-n)x] \, dx$$

$= 0$ if $m \neq n$.

$= \pi$ if $m = n$.

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PG TRB MATHEMATICS 2020 - 21

QUESTIONS - UNIT – III

FOURIER SERIES AND FOURIER INTEGRALS

1. The fourier integral f(x) is represented as,

A) f(x) = $\int_{-\infty}^{\infty}[A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x)]\, d\lambda;\ \forall\, \varkappa \in R$

B) f(x) = $\int_{c}^{\infty}[A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x)]\, d\lambda;\ \forall\, \varkappa \in R$

C) f(x) = $\int_{o}^{L}[A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x)]\, d\lambda;\ \forall\, \varkappa \in R$

D) f(x) = $\frac{1}{2}\int_{-\infty}^{\infty}[A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x)]\, d\lambda;\ \forall\, \varkappa \in R$

2. The fourier integral is useful for

A) Periodic function

B) Non-periodic function

C) Lograthemetic function

D) Discontinuous function

3. The fourier integral formula decomposition for

A) Periodic function into non-periodic function

B) Non-periodic function into periodic function

C) Non-periodic function into harmonic function

D) Harmonic function into periodic function

4. f(x)= e^{-kx} , x>0,K>0 then A(w)

A) 2 K/π (K^2+w^2)

B) 2 w/π (K^2+w^2)

C) W/π (K^2+w^2)

D) K/π (K^2+w^2)

5. If f(x) $\begin{cases} \frac{\pi}{2}; 0 < x < \pi \; then \; B\;(w) \\ \quad 0\;; x > \pi \end{cases}$

A) (1-cos w$\frac{\pi}{w}$)/w

B) (1-(coswπ/w0)

C) (r/2-(wwπ/w

D) 1+(coswπ/w

6. The fourier integral of f(x) = $\begin{cases} 1 \; |x| < 1 \\ 0 \; |x| < 1 \end{cases}$ is

A) $\int_0^\infty (2 \sin wx/\pi w) coszdw$

B) $\int_0^\infty (\sin wx/\pi w) coszdw$

C) $\int_0^\infty (2 \sin w/\pi w) coszdw$

D) $\int_0^\infty (2 \sin ww/\pi w) coszdw$

7. The fourier integral of f(r) = $\begin{cases} 2 \; |x| < 2 \\ 0 \; |x| > 2 \end{cases}$ is

A) $\int_0^\infty (4 \; \sin wx/\pi w) coswxdw$

B) $\int_0^\infty (4 \; \sin 2w/\pi w) coswxdx$

C) $\int_{-\infty}^\infty (4 \; \sin 2w/\pi w) coswxdx$

D) $\int_\infty^{-\infty} (4 \; \sin 2w/\pi w) coswxdx$

8. If A(w) is zero then given function is

A) Even

B) odd

C) Neither even nor odd

D) both a and b

9. If B (w) is zero in given function is

A) even

B) odd

C) Neither even nor odd

D) both a and b

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2020-2021

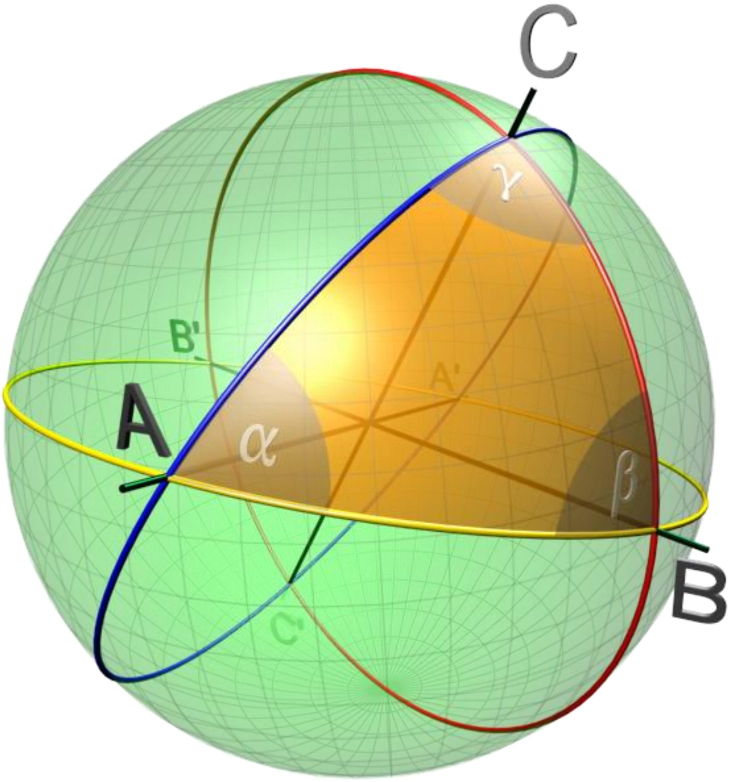
UNIT-4

Mathematics

Differential Geometry

TEACHER'S CARE ACADEMY
KANCHIPURAM

MATHEMATICS
UNIT – IV (Vol 1)



COMPETITIVE EXAM
FOR
PG-TRB 2020 – 21

SYLLABUS: MATHEMATICS

UNIT- IV

DIFFERENTIAL GEOMETRY

Curves in spaces - Serret-Frenet formulas - Locus of centers of curvature - Spherical curvature - Intrinsic equation – Helices - Spherical indicatrix surfaces – Envelope - Edge of regression – Developable surfaces associated to a curve - first and second fundamental forms - lines of curvature - Meusnieu's theorem - Gaussian curvature - Euler's theorem - Duplin's Indicatrix - Surface of revolution conjugate systems - Asymptritic lines - Isolmetric lines – Geodesics.

Theory of space curves.

Section : 1.2

Representation of space curves :-

Introduction

Differential geometry may be describes as the study of curves and surfaces. It is the branch of geometry which is treated with the help of Differential Calculus.

Differential Geometry is the study of properties of space curves and surfaces with the help of V.C. This geometry examines in more details the curves in space and surfaces, whereas the differential geometry of the plane curves deals with the tangent, normals, curvature, asymptotes, involutes, evolutes etc.

Generally differential Geometry deals with the properties of restricted portion of an geometric configuration where as algebraic geometry concern with the properties of the configuration as an whole.

In the theory of plane curves, a curve is usually specified either by means of a single equation or else by a parametric representation.

For Example,

1. a circle is a plane curve, the cartesian coordinate of (x, y) by the single eqn is

$$x^2+y^2 = a^2.$$

Then the parametric representation.

$$x = a\cos u, \quad y = a\sin u, \quad z=0.$$

2. Straight line:

A straight line in the space can be given in the equation $x_i = a_i + u b_i$ — (*)

where a_i and b_i are constant and atleast one of the $b_i \neq 0$. This equation represents a line passing through the point a_i with these direction cosine proportional to b_i.

Then ① can be written as

$$\frac{x_1 - a_1}{b_1} = \frac{x_2 - a_2}{b_2} = \frac{x_3 - a_3}{b_3} = \dots$$

3. Circular helix:

The parametric representation of the equation are

$$x_1 = a\cos u, \quad x_2 = a\sin u, \quad x_3 = bu.$$

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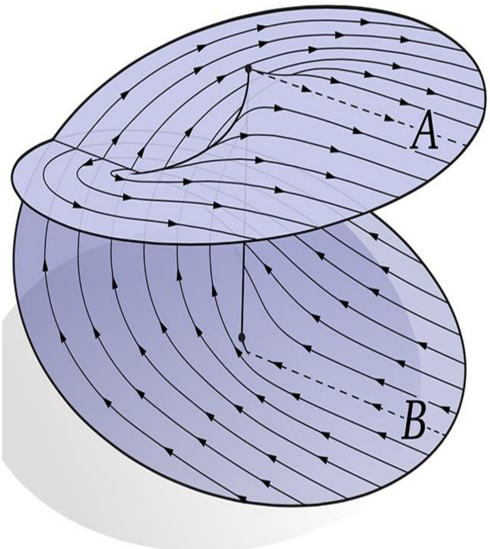
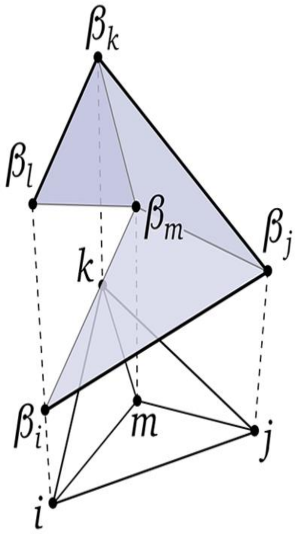
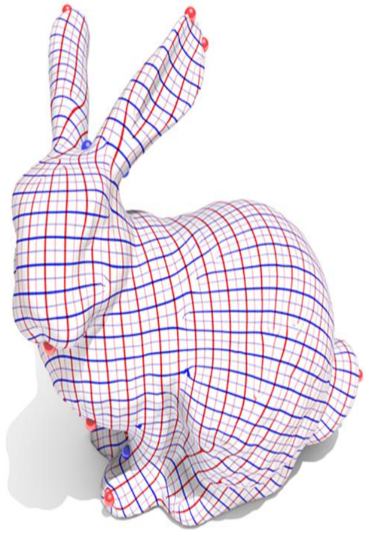
MATHEMATICS

UNIT – IV (Vol 2)







COMPETITIVE EXAM

FOR

PG-TRB 2020 – 21

SYLLABUS: MATHEMATICS

UNIT- IV

DIFFERENTIAL GEOMETRY

Curves in spaces - Serret-Frenet formulas - Locus of centers of curvature - Spherical curvature - Intrinsic equation – Helices - Spherical indicatrix surfaces – Envelope - Edge of regression – Developable surfaces associated to a curve - first and second fundamental forms - lines of curvature - Meusnieu's theorem - Gaussian curvature - Euler's theorem - Duplin's Indicatrix - Surface of revolution conjugate systems - Asymptritic lines - Isolmetric lines – Geodesics.

Section: 2:8 Helicoids.

Screw Motion :-

The surfaces obtained only by rotation about an axis in its plane such as spheres, cones and anchor ring. But there are surfaces which are generated not only by rotation alone but by a rotation followed by a translation. Such a motion is called screw motion.

Right helicoid :-

The surface generated by the screw motion of the x-axis about the z-axis is called a right helicoid.

Representation of a right helicoid :-

This is the helicoid generated by a straight line which meet the axis at right angles.

If x-axis as the generating line, it rotates about the z-axis and moves upwards.

Let $O'P$ be the translated position of the x-axis after rotating through an angle v.

Let $P(x, y, z)$ be any point.

Draw $PM \perp r$ to the xoy plane and.

let OM = u.

$\therefore x = u\cos v$, $y = u\sin v$, & $z = PM$ which is translated by the x-axis is proportion to the angle v of rotation.

Let $\frac{z}{v} = a$, a constant.

Hence the position vector of any point on the right helicoid is

$\bar{r} = (u\cos v, u\sin v, av)$

$\bar{r}_1 = (\cos v, \sin v, 0)$

$\bar{r}_2 = (-u\sin v, u\cos v, a)$.

$\bar{r}_1 \cdot \bar{r}_2 = -u\cos v\sin v + u\cos v\sin v + 0 = 0$.

$\therefore$ the parametric curves are orthogonal.

When u = constant c; then the eqn of the helicoid is

$\bar{r} = (c\cos v, c\sin v, av)$

which are the circular helices on the surface.

the parametric curves v = constant are the generators at the constant distance from the xoy plane

Unit - IV

Differential Geometry

1. In a helix the ratio κ / τ is equal to

(a) 1 (b) 0 (c) a constant (d) None of these

2. If R is the radius of spherical curvature, then

(a) $R^2 = \left| \frac{\vec{t}' \times \vec{t}''}{\kappa^2 \tau} \right|^2$ (b) $R = \left| \frac{\vec{t}' \times \vec{t}''}{\kappa^2 \tau} \right|^2$

(c) $R = 0$ (d) $R = |\vec{t}' \times \vec{t}''|^2$

3. If the fundamental coefficients L, M, N vanish everywhere on a surface then the surface is

(a) a sphere (b) a part of the plane

(c) a circle (d) an ellipse

4. If a surface is developable then its Gaussian curvature is

(a) zero (b) a constant (c) ∞ (d) none of these

5. The asymptotic lines of the paraboloid of revolution $z = x^2 + y^2$ is given by

(a) $du^2 + dv^2 = 0$ (b) $du^2 + u^2 dv^2 = 0$

(c) $u^2 du^2 + v^2 dv^2 = 0$ (d) None of these

6. The condition for the parametric curves to have conjugate direction is

(a) $M = 0$ (b) $N = 0$ (c) $M = N = 0$ (d) None of these

7. The condition for a curve on a surface is a geodesic of

(a) $V \frac{\partial T}{\partial \dot{u}} + U \frac{\partial T}{\partial \dot{v}} = 0$ (b) $V \frac{\partial T}{\partial \dot{u}} - U \frac{\partial T}{\partial \dot{v}} = 0$

(c) $U \frac{\partial T}{\partial \dot{u}} + V \frac{\partial T}{\partial \dot{v}} = 0$ (d) None of these.

8. The geodesics on right circular cylinder are

(a) right circular cones (b) itself

(c) circles (d) helix

9. The radius of spherical curvature R is

(a) $\rho^2 + \sigma^2 \rho'^2$ (b) $(\rho^2 + \sigma^2 \rho'^2)^{1/2}$ (c) $(\rho^2 + \sigma \rho'^2)^2$ (d) $\rho^2 - \sigma^2 \rho'^2$

10. C is a curve and S is a surface then C and S have three point contact at t_0 if

(a) $F'(t_0) = F''(t_0) = 0$ and $F'''(t_0) \neq 0$

(b) $F'(t_0) = 0$, $F''(t_0) \neq 0$ and $F'''(t_0) = 0$

(c) $F'(t_0) \neq 0$, $F''(t_0) \neq 0$ and $F'''(t_0) = 0$

(d) $F'(t_0) \neq 0$, $F''(t_0) = 0$ and $F'''(t_0) = 0$

11. The curvature of indicatrix is the ratio of the

(a) circular curvature to the screw curvature

(b) Screw curvature to the circular curvature of the curve

(c) Sperical curvature to the screw curvature of the curve

(d) Spherical curvature to the circular curvature of the curve

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2020-2021

Mathematics

Operations Research

UNIT-5

SYLLABUS: MATHEMATICS

UNIT- V

OPERATIONS RESEARCH

Linear programming - Simplex Computational procedure - Geometric interpretation of the simplex procedure - The revised simplex method - Duality problems - Degeneracy procedure - Peturbation techniques - integer programming - Transportation problem – Non-linear programming – The convex programming problem - Dyamic programming - Approximation in function space, successive approximations - Game theory - The maximum and minimum principle - Fundamental theory of games - queuing theory / single server and multi server models (M/G/I), (G/M/I), (G/G1/I) models, Erlang service distributions cost Model and optimization - Mathematical theory of inventory control - Feed back control in inventory management - Optional inventory policies in deterministic models - Storage models - Damtype models - Dams with discrete input and continuous output - Replacement theory – Deterministic Stochostic cases - Models for unbounded horizons and uncertain case - Markovian decision models in replacement theory - Reliability - Failure rates - System reliability - Reliability of growth models – Net work analysis - Directed net work - Max flowmin cut theorem - CPM-PERT - Probabilistic condition and decisional network analysis.

Unit - V

Operation Research.

- Origin during world war II. When the British military asked scientists to analyze military problems.
- The application of mathematics and scientific method to military applications was called operation research.
- It is a scientific approach to decision making that seeks to determines how best to operate a system under conditions of allocating scarce resources.

Formulation of operational linear programming problem.

Example - I.

- Consider a small manufacturer making two products A & B.
- Two resources R_1 & R_2 are required to make these products.
- Each unit of product A requires 1 unit of R_1 and 2 units of R_2 & B requires 1 unit of R_1 and 3 units of R_2.

(2)

- The manufacturer has 5 units of R_1 and 12 units of R_2 available.
- The manufacturer also makes a profit of
  - Rs. 6 per unit of product A sold &
  - Rs. 5 per unit of product B sold.

Let x_1, x_2 be the number of products A & B produced respectively (which makes the decision to produce).

Our objective is to maximize the profit.

So, $\max Z = 6x_1 + 5x_2$ (objective function).

But we have some restriction (limitedness) of on Resources R_1 & R_2.

A requires 1 unit of R_1 & 2 units of R_2.

B requires 1 unit of R_1 & 3 units of R_2.

But, the total availability of Resources R_1 & R_2 are 5 & 12 units respectively.

So, we have
$$\left.\begin{aligned} x_1 + x_2 &\le 5 \\ 2x_1 + 3x_2 &\le 12 \end{aligned}\right\} \text{(constraints)}.$$

x_1, x_2 denotes numbers so $x_1, x_2 \ge 0$.

↓

(non-negative constraints)

So the problem is formulated as :

Let x_1, x_2 denotes the decision variable denotes number of products A & B.

$$\max Z = 6x_1 + 5x_2.$$

Sub to :

$$x_1 + x_2 \leq 5$$

$$2x_1 + 3x_2 \leq 12.$$

& $x_1, x_2 \geq 0$ (non - negative constraints)

(or)

(non - negative restriction).

A linear programming problem has,

- a linear objective function
- linear constraints
- non - negativity constraints on all decision variable.

Steps to (write the LPP) Formulate LPP :-

I. Identify the decision variable.

II. Write the objective function.

III. State the constraints.

IV. Write the non - negative constraints for all decision variable.

Questions.

(1) Max $Z = 20x_1 + 18x_2$.

sub to $3x_1 + 3x_2 \leq 21$.

$4x_1 + 3x_2 \leq 24$.

$x_1, x_2 \geq 0$.

(a) has a unique sol. (b) has no solution (c) Infinitely many solutions. (d) None of the above.

(2) Max $Z = 6x_1 + 5x_2$.

Subject to $x_1 + x_2 \leq 5$

$3x_1 + 2x_2 \leq 12$.

$x_1, x_2 \geq 0$.

(a) has a unique sol. (b) has No Solution (c) Infinitely many sol (d) None.

(3) Max $Z = x_1 - x_2$ sub to, $x_1 - 2x_2 \leq 4$.

$2x_1 - x_2 \geq -2$

$x_1, x_2 \geq 0$.

(a) has a unique sol. (b) has unbounded sol. (c) has infinite sol (d) None.

(4) Min $Z = x_1 + 3x_2$ sub to $5x_1 + 4x_2 \geq 20$.

$3x_1 + 4x_2 \leq 24$.

$x_1, x_2 \geq 0$.

(a) has no solution (b) has unique sol. (c) has infinite Sol (d) None.

(5) min $Z = x_1 + 3x_2$ sub to $5x_1 + 4x_2 \geq 20$.

$3x_1 + 4x_2 \leq 24$.

$x_1, x_2 \geq 0$.

(a) $Z = 4$ (b) $Z = 5$ (c) $Z = 6$ (d) $Z = 4.5$.

(6) Max $z = 4x_1 + 3x_2$. subject to $2x_1 + 3x_2 \leq 8$.

$3x_1 + 2x_2 \leq 12$.

$x_1, x_2 \geq 0$

Write How many Intial basic feasible solution for this system.

(a) 4 (b) 6 (c) 5 (d) 2.

(7) Max $z = 10x_1 + 12x_2$. Sub to $3x_1 + 9x_2 \leq 5$

$4x_1 + 9x_2 \leq 6$.

$8x_1 + 8x_2 \leq 10$, $x_1, x_2 \geq 0$.

Write How many initial basic feasible solution for this system?

(a) 20 (b) 10 (c) 5 (d) 3.

(8) min $z = 8x_1 - 6x_2$ sub to, $x_1 - x_2 \leq 4$.

$4x_1 - 3x_2 \leq 8$

$x_1, x_2 \geq 0$.

(a) has bounded feasible region (b) has unbounded feasible region (c) None of the above.

(9) Max $z = 2x_1 + x_2$ sub to $x_1 + 2x_2 \leq 3$.

$x_2 \leq 5$

x_1 unrestricted

$x_2 \geq 0$.

(a) has No Sol. (b) has unique sol (c) Infinite no. of sol (d) None.

(10) An LPP has a unique solution and the feasible region is

(a) bounded (b) unbounded (c) No region exist (d) None.

(11) An LPP has a unique solution and the feasible region is

(a) Convex (b) Concave (c) Unbounded (d) None

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Mathematics

Functional Analysis

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2020-2021

UNIT-6

SYLLABUS: MATHEMATICS

UNIT- VI

FUNCTIONAL ANALYSIS

Banach Spaces - Definition and example - continuous linear transformations - Banach theorem - Natural embedding of X in X - Open mapping and closed graph theorem - Properties of conjugate of an operator - Hilbert spaces - Orthonormal bases - Conjugate space H - Adjoint of an operator – Projections 1^2 as a Hilbert space – 1^p space - Holders and Minkowski inequalities - Matrices – Basic operations of matrices - Determinant of a matrix - Determinant and spectrum of an operator - Spectral theorem for operators on a finite dimensional Hilbert space - Regular and singular elements in a Banach Algebra – Topological divisor of zero - Spectrum of an element in a Branch algebra - the formula for the spectral radius radical and semi simplicity.

Unit-VI

Banach Spaces.

Define Normed Linear Space.

A normed linear space is a linear space N in which to each vector x, there corresponds a real number denoted by $\|x\|$ called a norm of x. In such a way that

(i) $\|x\| \geq 0$ & $\|x\| = 0$ iff $x = 0$

(ii) $\|x+y\| \leq \|x\| + \|y\|$

(iii) $\|\alpha x\| = |\alpha| \, \|x\|$

Note:

The non-(-ve) real no. $\|x\|$ is to be thought of as the length of the vector x.

Prove that $\big|\|x\| - \|y\|\big| \leq \|x-y\|$:

Proof:

We know that

$$\|x\| = \|x-y+y\| \leq \|x-y\| + \|y\|$$

$$\|x\| - \|y\| \leq \|x-y\| \quad \text{---} (1)$$

$$-(\|x\| - \|y\|) = \|y\| - \|x\|$$

$$\leq \|y-x\| = \|(-1)(x-y)\|$$

$$= (-1)\|x-y\| \quad \text{---} (2)$$

From (1) & (2)

$$\big|\|x\| - \|y\|\big| \leq \|x-y\|$$

Hence proved.

But $\|x+M\| = \inf.\{\|x+m\| : m \in M\}$

$\therefore$ $x \in M$, then $x+M = y \in M$

$= \inf.\{\|y\| ; y \in M\}$

$\because$ M is a subspace containing the zero elt.

$\therefore$ $x+M = M \Rightarrow \|x+M\| = 0$.

Hence $\|x+M\| \geq 0$ & $\|x+M\| = 0 \Leftrightarrow x+M = M$

Hence condition ① is satisfied.

Let $x+M, y+M \in N/M$. Then by definition

$(x+M) + (y+M) = x+y+M$

$\|(x+yM) + (y+M)\| = \|(x+y)+M\|$

$= \inf.\{\|x+y+m\| ; m \in M\}$

$= \inf.\{\|(x+m_1) + (y+m_2)\| : m_1+m_2 \in M\}$

$\leq \inf \{\|x+m_1\| ; m_1 \in M\} +$

$\inf \{\|y+m_2\| ; m_2 \in M\}$

$= \|x+M\| + \|y+M\|$

Hence condition ② is satisfied.

W.K.T $\alpha(x+M) = \alpha x + M$

$\therefore$ $\|\alpha(x+M)\| = \|\alpha x + M\|$

$= \inf.\{\|\alpha x + M\| ; m \in M\}$

$= \inf.\{\|\alpha x + M\| ; m' \in M\}$.

$= \inf.\{|\alpha| \|x + M\| ; m' \in M\}$

$= |\alpha| \inf.\{\|x+m'\| ; m' \in M\}$

$= |\alpha| \|x+M\|$

Hence N/M is a normal Linear space.

Assume that N is complete.

Now we shall s.t N/M is complete.

Let $\{x_n + M\}$ be a Couchy seq. in N/M.

To prove $\{x_n + M\}$ is convergent it is sufficient to s.T it has a cgt. sub seq.

Choose a sub seq. as follows.

Let $y_1 \in x_1 + M$

Choose $y_2 \in x_2 + M$ s.t

$\|y_1 - y_2\| \leq \frac{1}{2}$,

Next choose y_3 in $x_3 + M$ s.t

$\|y_2 - y_3\| < \frac{1}{2^2}$.

Continuing in this way we get a sub seq.

$\{y_n\}$ in N s.t $\|y_n - y_{n+1}\| < \frac{1}{2^n}$.

If $m < n$ then $\|y_m - y_n\| < \frac{1}{2^{m-1}}$.

Assume $m \to \infty$, then $\frac{1}{2^{m-1}} \to 0$.

$\therefore \|y_m - y_n\| \to 0$ as $m, n \to \infty$.

$\because$ N is complete & seq $\{y_n\}$ is a Couchy's seq. in N then $\exists$ a vector y in N s.t $y_n \to y$. Then

$\|(x_n + M) - (y + M)\| = \|(x_n - y) + M\|$

$= \inf.\{\|(x_n + m) - y\| ; m \in M\}$

$\because y_n \in x_n + M$ then we've $\|y_n - y\|$.

$\because y_n \to y$ then $\|y_n - y\| \to 0$.

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UNIT-6

FUNCTIONAL ANALYSIS

QUESTIONS

1. If N is a normed linear space then the norm is a
   A) Continuous function on N
   B) Jointly continuous
   C) Unbounded
   D) None
2. If N is a normed linear space then
   A) $|\|x\| - \|y\|| > \|x - y\|$
   B) $|\|x\| - \|y\|| \leq \|x - y\|$
   C) $|\|x\| + \|y\|| \leq \|x - y\|$
   D) None
3. If N is a NLS every convergent sequence is a
   A) Continuous
   B) Bounded
   C) Cauchy sequence sequence
   D) None
4. Every Cauchy sequence in a NLS is
   A) Bounded
   B) Unbounded
   C) Constant
   D) None

5. A Cauchy sequence in a NLS is
   A) Convergent
   B) Need not be convergent
   C) Unbounded
   D) None
6. A complete NLS is called a
   A) Banach space
   B) Hilbert's space
   C) Vector space
   D) None
7. A NLS N is said to be separable if it has a countable
   A) Subset
   B) Separable subset
   C) Dense subset
   D) None
8. The NLS l_p , $1 \leq p < \infty$ are
   A) Separable
   B) Complete
   C) Compact
   D) None
9. The space l_∞ is
   A) Separable
   B) Not separable
   C) Complete
   D) None
10. Every complete subspace M of NLS N is
    A) Complete
    B) Closed
    C) Not separable
    D) None
11. If M is a closed linear subspace of a NLS the the quotient space N/M is a NLS with the norm of each coset x+M defined as $\|x + M\| = \inf\{\|x + m\| \ ; \ \ m \in M\}$ if N is a banch space then
    A) N+M is a NLS

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Mathematics

Complex Analysis

UNIT-7

PG TRB

2020-2021

SYLLABUS: MATHEMATICS

UNIT- VII

COMPLEX ANALYSIS

Introduction to the concept of analytic function - limits and continuity - analytic functions - Polynomials and rational functions elementary theory of power series – Maclaurin’s series – uniform convergence power series and Abel’s limit theorem - Analytic functions as mapping - conformality arcs and closed curves - Analytical functions in regions - Conformal mapping - Linear transformations – the linear group, the cross ratio and symmetry - Complex integration - Fundamental theorems - line integrals - rectifiable arcs - line integrals as functions of arcs - Cauchy’s theorem for a rectangle, Cauchy’s theorem in a Circular disc, Cauchy’s integral formula - The index of a point with respect to a closed curve, the integral formula - higher derivatives - Local properties of Analytic functions and removable singularities- Taylor’s theorem - Zeros and Poles - the local mapping and the maximum modulus Principle.

COMPLEX ANALYSIS

Introduction to the concept of analytic functions - limits and continuity - analytic functions - Polynomials and rational functions elementry theory of power series - Maclaurin's series - Uniform convergence power series and Able's limit theorem - Analytic functions as mapping - conformality arcs and closed curves - Analytical functions in regions - conformal mapping - linear transformations - the linear group, the cross ratio and symmetry - complex integration - fundamental theorems - line integrals - rectifiable arcs - line integrals as functions of arcs - Cauchy's theorem for rectangle, Cauchy's theorem in a circular disc, Cauchy's integral formula - The index of a point with respect to a closed curve, the integral formula - higher derivatives - local properties of analytic functions and removable singularities - Taylor's theorem - zero's and poles - the local mapping and the maximum modulus principle.

Algebra of Complex numbers:

Let $\mathbb{C} = \{x+iy \,/\, x, y \in \mathbb{R}\}$ be the set of all Complex numbers. Define

$$z_1 + z_2 = (x_1+iy_1) + (x_2+iy_2) = (x_1+x_2) + i(y_1+y_2)$$

and

$$z_1 \cdot z_2 = (x_1+iy_1)\cdot(x_2+iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

under these binary operations '+', '·', the set of all Complex numbers $\mathbb{C}$ becomes a field.

Also, by a known result a field over itself be a vector space. Thus $\mathbb{C}$ is a 1-dimensional vector space over $\mathbb{C}$.

Now, define a absolute value of the Complex number $z = x+iy$ by means of Euclidean geomentry.

i.e., $|z|$ is distance b/w the point z and the origin.

i.e., $|z| = |x+iy| = \sqrt{x^2+y^2}$

Properties:

(i) $\bar{z}$ denotes the conjugate of z, denote $\bar{z} = x - iy$, where $z = x+iy$.

Then $|\bar{z}| = \sqrt{x^2+y^2}$

Thus, $|z| = |\bar{z}|$

(ii) $z \cdot \bar{z} = (x+iy)\cdot(x-iy)$

$= x^2 + y^2$

$z \cdot \bar{z} = |z|^2 = |\bar{z}|^2$

(iii) $Re(z) = x \leq |x| \leq |z|$

$Im(z) = y \leq |y| \leq |z|$

Since, $x, y, |x|, |y|$ and $|z|$ are real numbers.

(iv) $z + \bar{z} = 2x$

$$x = \frac{z+\bar{z}}{2}$$

$$Re(z) = \frac{z+\bar{z}}{2}$$

$$z - \bar{z} = 2iy$$

$$y = \frac{z-\bar{z}}{2i}$$

$$Im(z) = \frac{z-\bar{z}}{2i}$$

(v) $\overline{z_1+z_2} = \overline{(x_1+x_2)+i(y_1+y_2)}$

$= (x_1+x_2) - i(y_1+y_2)$

$= (x_1 - iy_1) + (x_2 - iy_2)$

$= \bar{z}_1 + \bar{z}_2$

$\overline{z_1 z_2} = \overline{(x_1x_2 - y_1y_2) + i(x_1y_1 + x_2y_2)}$

$= (x_1x_2 - y_1y_2) - i(x_1y_2 + x_2y_1)$

$\bar{z}_1 = x_1 - iy_1$

$\bar{z}_2 = x_2 - iy_2$

$\bar{z}_1 \cdot \bar{z}_2 = (x_1 - iy_1)\cdot(x_2 - iy_2)$

$= (x_1x_2 - y_1y_2) - i(x_1y_2 + y_1x_2)$

$\therefore \overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

(vi) $|z_1z_2| = \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + y_1x_2)^2}$

$= \sqrt{x_1^2x_2^2 + y_1^2y_2^2 - 2x_1x_2y_1y_2 + x_1^2y_2^2 + y_1^2x_2^2 + 2x_1x_2y_1y_2}$

1. f(z) is analytic at z if

(a) $\frac{df}{dz} = 0$ (b) $\frac{df}{d\bar{z}} = 0$ (c) $\frac{df}{dx} = 0$ (d) $\frac{df}{dy} = 0$.

Ans:

2. If f(z) is single valued analytic with in and on a simple closed curve c, then $\int_c f(z)\,dz = 0$ is

(a) Morera's theorem (b) Liouville's theorem

(c) Fundamental theorem (d) Cauchy's integral theorem.

Ans:

3. Every singular point of f(z) which is not a pole is called as

(a) removable singularity of 2nd kind (b) isolated point.

(c) essential singularity (d) removable singularity of 1st kind.

Ans:

4. Poles of cot z are

(a) $z = \pi$ (b) $z = n\pi$ (c) $z = \pi/2$ (d) $z = \frac{n\pi}{2}$.

Ans:

5. The power series $\sum_{n=0}^{\infty} z^n$

(a) Converges in $|z|<1$ and diverges in $|z|\geq 1$

(b) Converges in $|z|>1$ and diverges in $|z|<1$.

(c) Converges in $|z|=1$ and diverges in $|z|>1$.

(d) Oscillates between -1 and 1.

Ans:

6. If a power series in z converges at $z=z_1$, then it converges absolutely in the open disk $|z|<|z_1|$ is

(a) Cauchy's theorem (b) Abel's theorem

(c) Laurent's theorem (d) Rouches theorem.

Ans:

7. The fixed points of $w = \frac{z}{2-z}$ are

(a) 0,2 (b) 0,0 (c) 0,1 (d) 1,2.

Ans:

8. A bilinear transformation having only one fixed point is called

(a) Parbola (b) hyperbolic (c) elliptic (d) none of these

Ans:

9. The cross ratio (z_1, z_2, z_3, z_4) is real iff the four points z_1, z_2, z_3, z_4 lie on a

(a) straight line (b) circle (c) rectangle (d) none of these

Ans:

10. The zeros of $f(z) = \sin z - \cos z$ are

(a) $z = \pi/4$ (b) $z = n\pi - \pi/4$ (c) $z = n\pi + \pi/4$ (d) $z = n\pi$

Ans:

11. $\int_C f(z)dz$ is equal to.

(a) $2\pi i f(a)$ (b) $2\pi i \, Im f(a)$ (c) $2\pi i \, Res f(a)$ (d) $-2\pi i \, Res f(a)$

Ans:

12. If $f(z)$ is analytic with in and on the circle $|z-a|=r$, then

(a) $|f^n(a)| \le \frac{M}{r^n}$ (b) $|f^n(a)| \le \frac{M^n}{r^n}$ (c) $|f^n(a)| \ge \frac{m^n}{r^n}$ (d) $|f^n(a)| \le \frac{Mn!}{r^n}$.

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MATHEMATICS

Differential Equations

UNIT-8

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SYLLABUS: MATHEMATICS

UNIT- VIII

DIFFERENTIAL EQUATIONS

Linear differential equation - constant co-efficients - Existence of solutions – Wrongskian - independence of solutions - Initial value problems for second order equations - Integration in series - Bessel’s equation - Legendre and Hermite Polynomials - elementary properties - Total differential equations - first order partial differential equation - Charpits method.

Unit - VIII

Differential Equations.

Introduction:

Consider differential equations of order higher than one. In these differential equations the dependent variable and its derivatives appear only in the first degree, and are not multiplied together, their coefficients are constants or the functions of x. The general form of the equation is

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + P_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_n y = X \quad \text{—— ①}$$

where $P_1, P_2, \ldots, P_n$ and X are constants or functions of x. We shall use the following two results:

Result 1: If $y = f(x)$ is the general solution of

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + P_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots + P_n y = 0 \quad \text{—— ②}$$

and $y = F(x)$ is a general solution of eqn ① then

$$y = f(x) + F(x).$$

is a general solution of eqn ①.

Result 2: If $y = y_k$, $k = 1, 2, \ldots, n$ are the solutions of eqn (2) then

$$y = C_1 y_1 + C_2 y_2 + \cdots + C_n y_n.$$

where $C_1, C_2, \ldots, C_n$ are arbitrary constants; is also solution of eqn ②.

Result 1 describes the method of finding the general solution equation 2. First find the general solution of the eqn (2) and call it $y = f_1(c_1, c_2, \ldots, c_n, x)$. Then find the general solution of eqn ① which does not contain any arbitrary constants. Call it $y = f_2(x)$. Then $y = f_1 + f_2$ is the general solution of the eqn ①. f_1 is known as the Complementary function (denoted by CF) and f_2 is known as the particular integral (denoted by PI).

In rest, D stands for $\frac{d}{dx}$. With this notation, the eqn ② and ① can be written as

$$(D^n + P_1 D^{n-1} + \ldots + P_n) y = 0 \longrightarrow ③$$

and $$(D^n + P_1 D^{n-1} + \ldots + P_n) y = X \longrightarrow ④.$$

<u>Symbolic</u> <u>Operator</u>:

We have

$$(D - m_1) y = Dy - m_1 y = \frac{dy}{dx} - m_1 y.$$

$(D-m_1)(D-m_2)y$ is defined as the expression obtained on operating $(D-m_1)$ on $\left(\frac{dy}{dx} - m_2 y\right)$.

i.e $$\frac{d^2y}{dx^2} - (m_1 + m_2)\frac{dy}{dx} + m_1 m_2 y.$$

where m_2 is constant. The operation $(D-m_2)(D-m_1)y$ is equivalent to above expression where m_1 is constant. That is, if m_1 and m_2 are both constants, the expression $(D-m_1)(D-m_2)y$ and $(D-m_2)(D-m_1)y$ are equivalent. Thus, the expression is independent of the order of the operational factors.

Method For Finding CF :

consider equation (3) and let it be equivalent to

$$(D-m_1)(D-m_2)\dots(D-m_n)y=0 \longrightarrow (5).$$

Then
$$\left.\begin{aligned}(D-m_1)y&=0.\\(D-m_2)y&=0.\\&\vdots\\(D-m_n)y&=0\end{aligned}\right\}$$

is also solution of equation (5)

Consider the equation

$$(D-m_k)y=0.$$

$$\Rightarrow \frac{dy}{dx}=m_k y.$$

$$\Rightarrow \frac{dy}{dx}=m_k\,dx.$$

which on integration gives

$$y=c_k e^{m_k x},\ k=1,2,\dots,n.$$

Then

$$y=\sum_{k=1}^{n} c_k e^{m_k x} \longrightarrow (6)$$

is a solution of eqn (5). If $m_1, m_2, \dots, m_n$ are distinct, the functions $e^{m_1 x}, e^{m_2 x}, \dots, e^{m_n x}$ are linearly independent, and hence, eqn (6) is the general solution of eqn (5). $m_1, m_2, \dots, m_n$ are the roots of the eqn.

$$m^n+P_1 m^{n-1}+\dots+P_n=0. \longrightarrow (7)$$

The eqn (7) is known as the Auxillary Equation (AE).

Unit - VIII

Differential Equations

1. Which Method can be used to solve this equation $f(x)dx + F(y)dy$?

(a) Variable separable (b) Homogenous Equation
(c) Non-Homogeneous equation (d) None

2. The Solution of $\frac{dy}{\sqrt{1-y^2}}$ after integration is

(a) $\sin^{-1} y = c$ (b) $\cos^{-1} y = 0$
(c) $\sin^{-1} y = 0$ (d) $\cos^{-1} y = c$.

3. The Solution of $x\,dy = y\,dx$ after integration is

(a) $x = cy$ (b) $x - y = c$ (c) $x^2 + y^2 = 0$ (d) $x^2 - y^2 = 0$

4. The Solution of $\sinh x\,dx = 0$ after integration is

(a) $\cosh x = c$ (b) $\cos x = c$ (c) $\cosh^{-1} x = c$ (d) $\cosh x = 0$

5. Which method could be used to solve this equation $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

(a) Homogeneous Equation (b) Variable separable
(c) Non-Homogeneous Equation (d) Linear Equation

6. If $y = vx$ then $\frac{dy}{dx} =$

(a) $v + x\frac{dv}{dx}$ (b) $v - x\frac{dv}{dx}$ (c) $v^2 + x^2\frac{dv}{dx}$ (d) $v^2 - x^2\frac{dv}{dx}$

7. The solution of $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$ after integration is

(a) $\tan^{-1}x - \cot^{-1}y = c$ (b) $\tan^{-1}x + \tan^{-1}y = c$

(c) $\tan^{-1}x + \cot^{-1}y = C$ (d) $-\cot^{-1}x + \cot^{-1}y = c$.

8. When the dependent variable and its derivatives occur only in the first degree a differential Equation is said to be

(a) linear (b) non-linear (c) homogeneous Equation (d) none

9. Integrating factor is one which changes a differential Equation

(a) An exact differential Equation (b) An approximate differential Eqn

(c) A mere differential Equation (d) None of these.

10. The integrating factor of $\frac{dy}{dx} + Py = Q$ where P & Q are functions of x only is

(a) $e^{\int P dx}$ (b) $e^{\int Q dy}$ (c) $e^{\int Q dx}$ (d) $e^{\int P dy}$

11. Which type can be used to solve the equation $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$

(a) Bernoulli's equation (b) Linear equation

(c) Homogeneous equation (d) variable separable.

12. The solution of $e^x dx + e^y dy = 0$ after integration is

(a) $e^x + e^y = c$ (b) $xe^x + ye^y = c$ (c) $e^x + e^y = 0$ (d) $xe^x + ye^y = 0$

13. The solution of $\operatorname{cosec}x \cot x \, dx = 0$ after integration is

(a) $-\operatorname{cosec}x = C$ (b) $\sec x = c$ (c) $\cot x = c$ (d) $\operatorname{cosec}x \cot x = c$

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Mathematics

STATISTICS-I

Your Success is Our Goal...

UNIT-9

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SYLLABUS: MATHEMATICS

UNIT- IX

STATISTICS - I

Statistical Method - Concepts of Statistical population and random sample - Collections and presentation of data - Measures of location and dispersion - Moments and shepherd correction – cumulate - Measures of skewness and Kurtosis - Curve fitting by least squares – Regression - Correlation and correlation ratio - rank correlation - Partial correlation - Multiple correlation coefficient – Probability Discrete - sample space, events - their union - intersection etc. - Probability classical relative frequency and axiomatic approaches - Probability in continuous probability space - conditional probability and independence - Basic laws of probability of combination of events - Baye's theorem – probability functions - Probability density functions - Distribution function - Mathematical Expectations – Marginal and conditional distribution - Conditional expectations.

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UNIT IX STATISTICS I

STATISTICAL METHODS

CONCEPT OF STATISTICAL POPULATION AND RANDOM SAMPLE

Populations

In statistics the term "population" has a slightly different meaning from the one given to it in ordinary speech. It need not refer only to people or to animate creatures - the population of Britain, for instance or the dog population of London. Statisticians also speak of a population of objects, or events, or procedures, or observations, including such things as the quantity of lead in urine, visits to the doctor, or surgical operations. A population is thus an aggregate of creatures, things, cases and so on.

Although a statistician should clearly define the population he or she is dealing with, they may not be able to enumerate it exactly. For instance, in ordinary usage the population of England denotes the number of people within England's boundaries, perhaps as enumerated at a census. But a physician might embark on a study to try to answer the question "What is the average systolic blood pressure of Englishmen aged 40-59?" But who are the "Englishmen" referred to here? Not all Englishmen live in England, and the social and genetic background of those that do may vary. A surgeon may study the effects of two alternative operations for gastric ulcer. But how old are the patients? What sex are they? How severe is their disease? Where do they live? And so on. The reader needs precise information on such matters to draw valid inferences from the sample that was studied to the population being considered. Statistics such as averages and standard deviations, when taken from populations are referred to as population parameters. They are often denoted by Greek letters: the population mean is denoted by μ(mu) and the standard deviation denoted by σ (low case sigma)

Samples

A population commonly contains too many individuals to study conveniently, so an investigation is often restricted to one or more samples drawn from it. A well-chosen sample will contain most of the information about a particular population parameter but the relation between the sample and the population must be such as to allow true inferences to be made about a population from that sample.

Consequently, the first important attribute of a sample is that every individual in the population from which it is drawn must have a known non-zero chance of being included in it; a natural suggestion is that these chances should be equal. We would like the choices to be made independently; in other words, the choice of one subject will not affect the chance of other subjects being chosen. To ensure this we make the choice by means of a process in which chance alone operates, such as spinning a coin or, more usually, the use of a table of random numbers.

Before drawing a sample the investigator should define the population from which it is to come. Sometimes he or she can completely enumerate its members before beginning analysis - for example, all the livers studied at necropsy over the previous year, all the patients aged 20-44 admitted to hospital with perforated peptic ulcer in the previous 20 months. In retrospective studies of this kind numbers can be allotted serially from any point in the table to each patient or specimen. Suppose we have a population of size 150, and we wish to take a sample of size five. Contains a set of computer generated random digits arranged in groups of five. Choose any row and column, say the last column of five digits. Read only the first three digits, and go down the column starting with the first row. Thus we have 265, 881, 722, etc. If a number appears between 001 and 150 then we include it in our sample. Thus, in order, in the sample will be subjects numbered 24, 59, 107, 73, and 65. If necessary we can carry on down the next column to the left until the full sample is chosen.

The use of random numbers in this way is generally preferable to taking every alternate patient or every fifth specimen, or acting on some other such regular plan. The regularity of the plan can occasionally coincide by chance with some unforeseen regularity in the presentation of the material for study - for example, by hospital appointments being made from patients from certain practices on certain days of the week, or specimens being prepared in batches in accordance with some schedule.

As susceptibility to disease generally varies in relation to age, sex, occupation, family history, exposure to risk, inoculation state, country lived in or visited, and many other genetic or environmental factors, it is advisable to examine samples when drawn to see whether they are, on average, comparable in these respects. The random process of selection is intended to make them so, but sometimes it can by chance lead to disparities. To guard against this possibility the sampling may be stratified. This means that a framework is laid down initially, and the patients or objects of the study in a random sample are then allotted to the compartments of the framework. For instance, the framework might have a primary division into males and females and then a secondary division of each of those categories into five age groups, the result being a framework with ten compartments. It is then important to bear in mind that the distributions of the categories on two samples made up on such a framework may be truly comparable, but they will not reflect the distribution of these categories in the population from which the sample is drawn unless the compartments in the framework have been designed with that in mind. For instance, equal numbers might be admitted to the male and female categories, but males and females are not equally numerous in the general population, and their relative proportions vary with age. This is known as stratified random sampling. For taking a sample from a long list a compromise between strict theory and practicalities is known as a systematic random sample. In this case we choose subjects a fixed interval apart on the list, say every tenth subject, but we choose the starting point within the first interval at random.

COLLECTIONS AND PRESENTATION OF DATA

Collection of data

Introduction

The basic problem of statistical enquiry is to collect facts and figures relating to a particular phenomenon under study, whether the enquiry is in business, economic or social science. The investigator is the person who conducts the statistical enquiry. He is a trained and efficient statistician. He or the statistician counts for measures the characteristic under study for further statistical analysis. The respondents are the persons from whom the information is collected. The statistical units are the items on which the measurement is taken. Collection of data is the process of enumeration together with the proper recording of results. The success of an enquiry is based upon the proper collection of data.

Primary and secondary data

Statistical data may be classified as primary and secondary. Primary data are those which are collected for the first time and they are original in character. If an individual or an office collects the data to study a particular problem, the data are the raw materials of the enquiry. They are primary data collected by the investigator himself to study any particular problem.

Objective questions –Unit IX

1. Sample is a sub-set of:

A) Population B) Data

C) Set D) Distribution

2. List of all the units of the population is called:

A) Random sampling B) Bias

C) Sampling frame D) Probability sampling

3. Any measure of the population is called:

A) Finite B) Parameter

C) Without replacement D) Random

4. If all the units of a population are surveyed, it is called:

A) Random sample B) Random sampling

C) Sampled population D) Complete enumeration

5. Probability distribution of a statistics is called:

A) Sampling B) Parameter

C) Data D) Sampling distribution

6. The difference between a statistic and the parameter is called:

A) Probability B) Sampling error

C) Random D) Non-random

7. Standard deviation of sampling distribution of a statistic is called:

A) Serious error B) Dispersion

C) Standard error D) Difference

8. A distribution formed by all possible values of a statistics is called

A) Binomial distribution B) Hypergeometric distribution

C) Normal distribution D) Sampling distribution

9. In probability sampling, probability of selecting an item from the population is known and is:

A) Equal to zero B) Non zero

C) Equal to one D) All of the above

10. A population about which we want to get some information is called:

A) Finite population B) Infinite population

C) Sampling population D) Target population

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MATHEMATICS

STATISTICS-II

UNIT-10

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SYLLABUS: MATHEMATICS

UNIT- X

STATISTICS-II

Probability distributions – Binomial, Poisson, Normal, Gama, Beta, Cauchy, Multinomial Hypergeometric, Negative Binomial - Chehychev's lemma (weak) law of large numbers - Central limit theorem for independent identical variates, Standard Errors - sampling distributions of t, F and Chi square - and their uses in tests of significance - Large sample tests for mean and proportions - Sample surveys - Sampling frame - sampling with equal probability with or without replacement - stratified sampling - Brief study of two stage systematic and cluster sampling methods - regression and ratio estimates - Design of experiments, principles of experimentation - Analysis of variance - Completely randomized block and latin square designs.

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UNIT - X - STATISTICS - II

PROBABILITY DISTRIBUTIONS

Types of Theoretical Probability distributions

The following are the two types of Theoretical distributions:

1. Discrete distribution 2. Continuous distribution

Discrete distribution

The binomial and Poisson distributions are the most useful theoretical distributions for discrete variables.

Continuous distribution

The binomial and Poisson distributions discussed in the previous chapters are the most useful theoretical distributions for discrete variables. In order to have mathematical distributions suitable for dealing with quantities whose magnitudes vary continuously like weight, heights of individual, a continuous distribution is needed. Normal distribution is one of the most widely used continuous distributions.

Normal distribution is the most important and powerful of all the distribution in statistics. It was first introduced by De Moivre in 1733 in the development of probability. Laplace (1749-1827) and Gauss (1827-1855) were also associated with the development of Normal distribution.

CHAPTER: 1

Bernoulli's Distribution

It is discovered by a Swiss Mathematician James Bernoulli (1654-1705) for a trial which has only two outcomes viz. a success with probability p and a failure with probability q= 1 – *p*.

Definition

A random variable X is said to follow a Bernoulli distribution if its probability mass function is given by

$$P(X=x) = \begin{cases} p^x q^{1-x} & x = 0,1 \\ 0 & otherwise \end{cases}$$

Characteristics of Bernoulli distribution

i. Number of trials is one

ii. $q = 1 - p$

iii. Constants of the distributions

iv. (i) mean = p (ii) variance = pq (iii) standard deviation = √pq

Chapter: 2

Binomial distribution

Binomial distribution was discovered by James Bernoulli (1654_1705) in the year 1700 and was first published posthumously in 1713, eight years after his death.

A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities p and q respectively, is called a Bernoulli trial.

Some examples of Bernoulli trials are:

(i) Tossing of a coin (Head or tail)

(ii) Throwing of a die (getting even or odd number)

Consider a set of n independent Bernoullian trails (*n* being finite) in which the probability '*p*' of success in any trial is constant , then $q = 1-p$, is the probability of failure. The probability of *x* successes and consequently

(*n*–*x*) failures in n independent trials, in a specified order (say) SSFSFFFS….FSF is given in the compound probability theorem by the expression

P(SSFSFFFS…..FSF) = P(S)P(S)P(F)P(S)x…….xP(F)P(S)P(F)

p.p.qp………………q.p.q

p.p. p.p. ……q.q.q.q.q.q

{x factors} {(n–x) factors}

$p^x q^{(n-x)}$

x successes in n trials can occur in ^nC_x ways and the probability for each of these ways is same namely $p^x\, q^{n-x}$.

The probability distribution of the number of successes, so obtained is called the binomial probability distribution and the binomial expansion is $(q + p)^n$

Definition

A random variable *X* is said to follow binomial distribution with parameter *n* and *p*, if it assumes only non- negative value and its probability mass function in given by

$$P\,(x = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x} & x = 0,1,2,\ldots..n; q = 1-p \\ 0, otherwise & \end{cases}$$

Note

Any random variable which follows binomial distribution is known as binomial variate i.e $X \sim B(n,p)$ is a binomial variate.

The Binomial distribution can be used under the following conditions:

1. The number of trials '*n*' finite

OBJECTIVE QUESTIONS:

Chapter: 14 **Introduction to Experimental Designs** and **Analysis of variance**

1. How many dependent variables does a two-way ANOVA have?

A) One

B) Two

C) Three

D) Four

2. What would the levels of the independent variables be for a two-way ANOVA investigating the effect of four different treatments for depression and gender?

A) 4 and 1

B) 4 and 4

C) 4 and 2

D) 6

3. How many independent variables were used and how were they measured in a three-way independent ANOVA?

A) Three independent variables all measured using the same entities

B) Three independent variables all measured using different entities

C) One independent variable (with three levels) measured using the same entities

D) One independent variable (with three levels) measured using different entities

4. Imagine we conducted a study that found that pedestrians were more likely to give money to a street beggar if the beggar had a cute and hungry-looking dog with them, and this effect was identical for both male and female pedestrians. If we calculated the difference between men and women in the no dog condition and plotted this value against the difference between men and women in the dog condition, which of the following values is most likely to represent the gradient of our graph?

A) 22.7

B) 33.8

C) 1

D) 0

5. Imagine we conducted a three-way independent ANOVA) How many sources of variance would we have?

A) 3

B) 7

C) 8

D) 4

6. Which test is applied to Analysis of Variance (ANOVA)?

A) t test

B) z test

C) F test

D) χ^2 test

7. Analysis of covariance is:

A) A statistical technique that can be used to help equate groups on specific variables

B) A statistical technique that can be used to control sequencing effects

C) A statistical technique that substitutes for random assignment to groups

D) Adjusts scores on the independent variable to control for extraneous variables

8. To determine whether noise affects the ability to solve math problems, a researcher has one group solve math problems in a quiet room and another group solve math problems in a noisy room. The group solving problems in the noisy room completes 15 problems in one hour and the group solving problems in the quiet room completes 22 problems in one hour. In this experiment, the independent variable is ____________ and the dependent variable is _____________.

A) The number of problems solves; the difficulty of the problems

B) The number of problems solved; the noise level in the room

C) The noise level in the room; the number of problems solved

D) The noise level in the room; the difficulty of the problems

9. The group that receives the experimental treatment condition is the _____.

A) Experimental group | B) Control group

C) Participant group | D) Independent group

10. The group that does not receive the experimental treatment condition is the ________.

A) Experimental group | B) Control group

C) Treatment group | D) Independent group

11. Which of the following could be used for randomly assigning participants to groups in an experimental study?

A) Split-half (e.g., first half versus second half of a school directory)

B) Even versus odd numbers

C) Use a list of random numbers or a computer randomization program

D) Let the researcher decide which group will be the best

12. A cell is a combination of two or more ____ in a factorial design.

A) Research designs | B) Research measurements

C) Dependent variables | D) Independent variables

MATHEMATICS